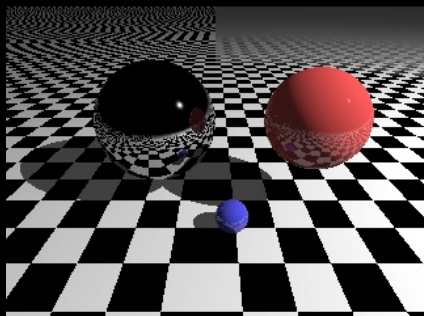
CSE168: Rendering Algorithms
Sampling and Anti-Aliasing



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Todays Menu

- What is sampling
- Aliasing
- Signal/Fourier analysis
- Sampling theorem
- Anti-aliasing - super sampling
- Sampling patterns
- Adaptive sampling

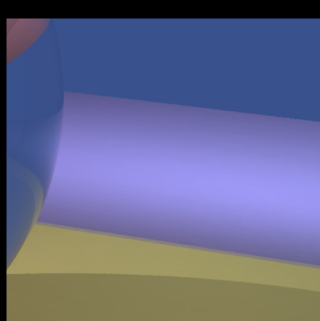
Sampling

- In general: Sampling of any function or signal
- Today: The process of tracing rays to decide the value of the pixels in an image

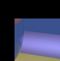
Why Sampling

- Digital representation (image) of a continuous signal (the incoming light).

A Pixel

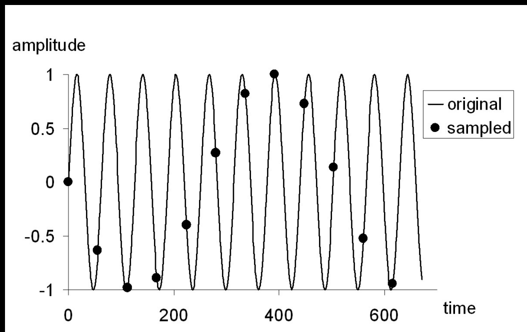


A Pixel

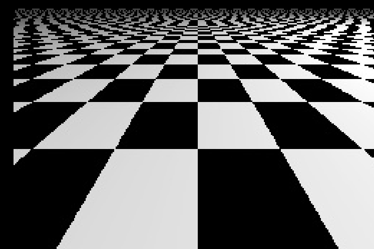


- How can we sample this?
- Is it possible to analyticallly compute?
- Is it possible to perfecty sample a pixel?
- Under what circumstances can we sample and reconstruct a pixel / signal perfectly?

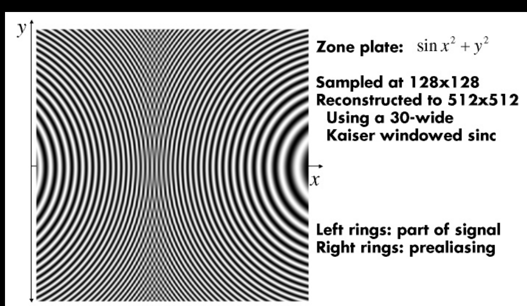
Aliasing Example 1D



Aliasing



Aliasing



Aliasing Artifacts

- Jaggies
- Moire
- Temporal aliasing
  - ⋆ Flickering small objects and highlights

Signal Analysis

- An image is a signal
- Our goal is to compute the image / signal

Fourier Analysis

$$F(\omega) = \int_{-\infty}^{\infty} f(x) e^{-i\omega x}\, dx$$

- Fourier transform $\rightarrow$ spectral representation of a signal as a sum of sines and cosines
- All functions have two representations
  - ⋆ Spatial domain: normal
  - ⋆ Frequency domain: spectral
- The Fourier transform converts between the spatial and frequency domains.

Fourier Analysis

Spatial Domain $\Longrightarrow F(\omega)=\int_{-\infty}^{\infty} f(x)e^{-i\omega x}\,dx \Longrightarrow$ Frequency Domain

Spatial Domain $\Longleftarrow f(x)=\frac{1}{2\pi}\int_{-\infty}^{\infty} F(\omega)e^{i\omega x}\,d\omega \Longleftarrow$ Frequency Domain

Convolution

$$f(x)*g(x)=\int_{-\infty}^{\infty} f(t)g(x-t)\,dt$$

Same as multiplication in the frequency domain.

Frequency Domain Convolution

$$F(\omega)*G(\omega)=\int_{-\infty}^{\infty} F(t)G(\omega-t)\,dt$$

Same as multiplication in the spatial domain.

Sampling

The Shah function

$$III_T(t)=\sum_{n=-\infty}^{\infty}\delta(x-nT)$$

Sampling of $f(x)\rightarrow f(x)III_T(t)$

Sampling Frequency Domain

The Fourier transform of the Shah function

$$III_{\omega_0}(\omega)=\frac{\kappa_T}{\kappa}\sum_{k=-\infty}^{\infty}\delta(\omega-k\omega_0)$$

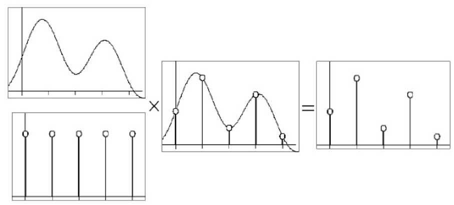
where $\omega_0=\frac{2\pi}{T}$

Sampling of $f(x)\rightarrow G(\omega)=F(\omega)*III_{\omega_0}(\omega)$

Sampling Frequency Domain

$$\begin{aligned}
G(\omega) &= F(\omega)*S(\omega)\\
&= F(\omega)*\frac{\kappa_T}{\kappa}III_{\omega_0}(\omega)\\
&= F(\omega)*\frac{\kappa_T}{\kappa}\sum_{k=-\infty}^{\infty}\delta(\omega-k\omega_0)\\
&= \frac{\kappa_T}{\kappa}\sum_{k=-\infty}^{\infty}F(\omega-k\omega_0)
\end{aligned}$$

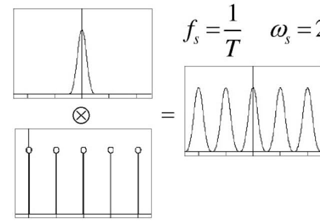
Sampling Spatial Domain



$$\mathrm{III}(x) = \sum_{n=-\infty}^{n=\infty} \delta(x - nT)$$

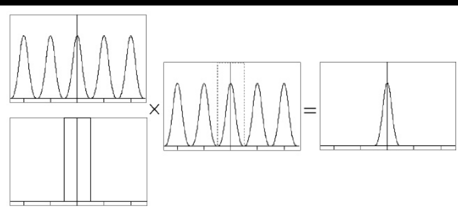
Sampling Frequency Domain

$$f_s = \frac{1}{T} \qquad \omega_s = 2\pi f_s = \frac{2\pi}{T}$$



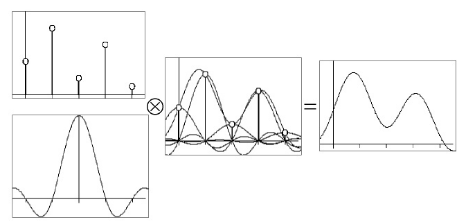
$$\mathrm{III}(\omega) = \sum_{n=-\infty}^{n=\infty} \delta(\omega - n\omega_s)$$

Reconstruction: Frequency Domain



$$\mathrm{II}(x) = \begin{cases} 1 & |x| \le \frac{1}{2} \\ 0 & |x| > \frac{1}{2} \end{cases}$$

Reconstruction: Spatial Domain



$$\mathrm{sinc}\, x = \frac{\sin \pi x}{\pi x}$$

Sampling Theorem

A signal (bandlimited function) can be reconstructed if it is sampled at twice the maximal frequency of the signal.

This sampling frequency is called the Nyquist Frequency.

Reconstruction

The Sinc filter

$$sinc(x) = \frac{sin(\pi x)}{\pi x}$$

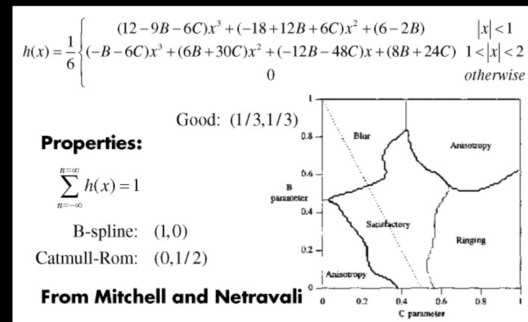
Problems:

- Infinite support
- Ringing

Reconstruction Filters

- Box filter
- B-spline
- Catmull-Rom
- Gauss filter
- Mitchell-Netravali

Michell-Netravali Cubic Filter



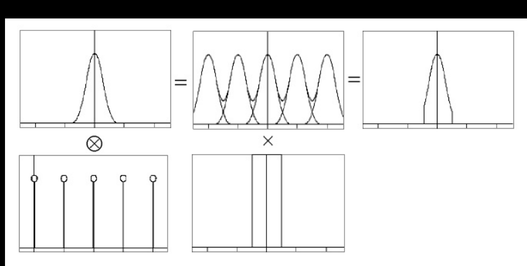
Sampling in Practice

1. Sampling
2. Reconstruction / filtering
3. Resampling

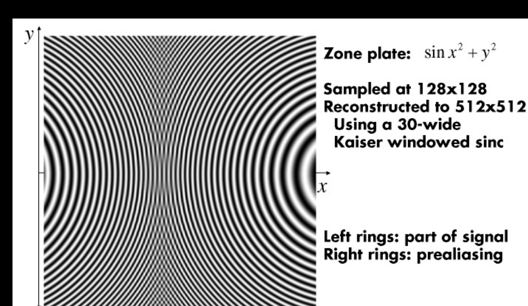
Anti-Aliasing

- The art of preventing aliasing

Undersampling



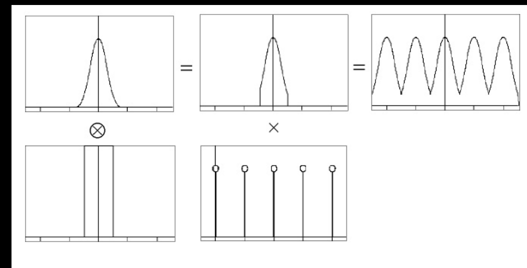
Zone Plate



Anti-Aliasing Techniques

- Prefiltering
- Supersampling
- Stochastic sampling

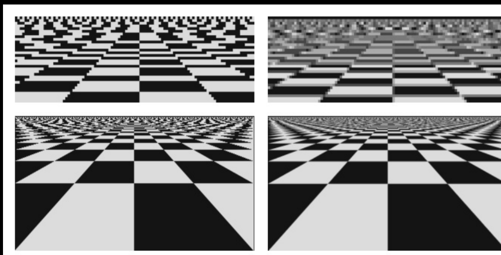
Prefiltering



Uniform Supersampling

- Higher frequency sampling
- Copies of signal spaced further apart in frequency domain

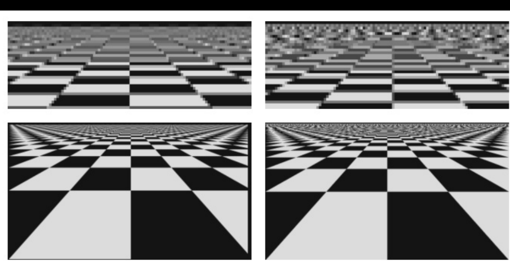
Uniform Supersampling



Point **4x4 Supersampled**

Checkerboard sequence by Tom Duff

Analytic vs. Supersampling



Exact Area **4x4 Supersampled**

Sampling Patterns

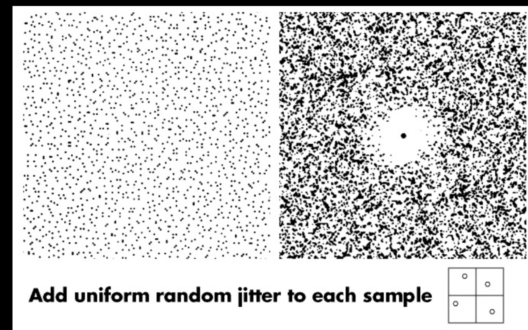
- Uniform grid
- Hexagonal
- Quincunx

Stochastic Sampling Patterns

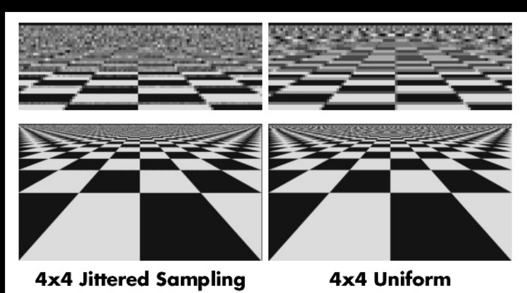
- Poisson
- Jittered sampling
- Poisson disk

We are less sensitive to noise than coherent aliasing.

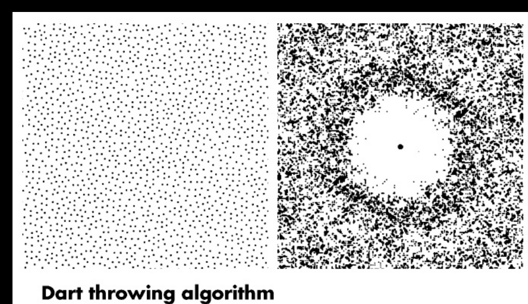
Jittering



Jittering vs. Supersampling



Poisson Disk



Adaptive Sampling

- Supersampling is expensive
- Every sample is precious
- Sample where it matters

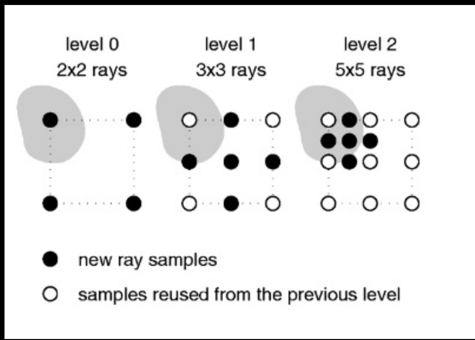
Adaptive Sampling

- Detect difference / contrast

Contrast function:

$$c = \frac{|r_2 - r_1|}{r_2 + r_1}$$

Adaptive Sampling



Next time

- Tone mapping and more