

# Irradiance Caching Methods

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CSE 168

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# Outline

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- Introduction
  - What is indirect illumination?

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- What is Irradiance Caching?

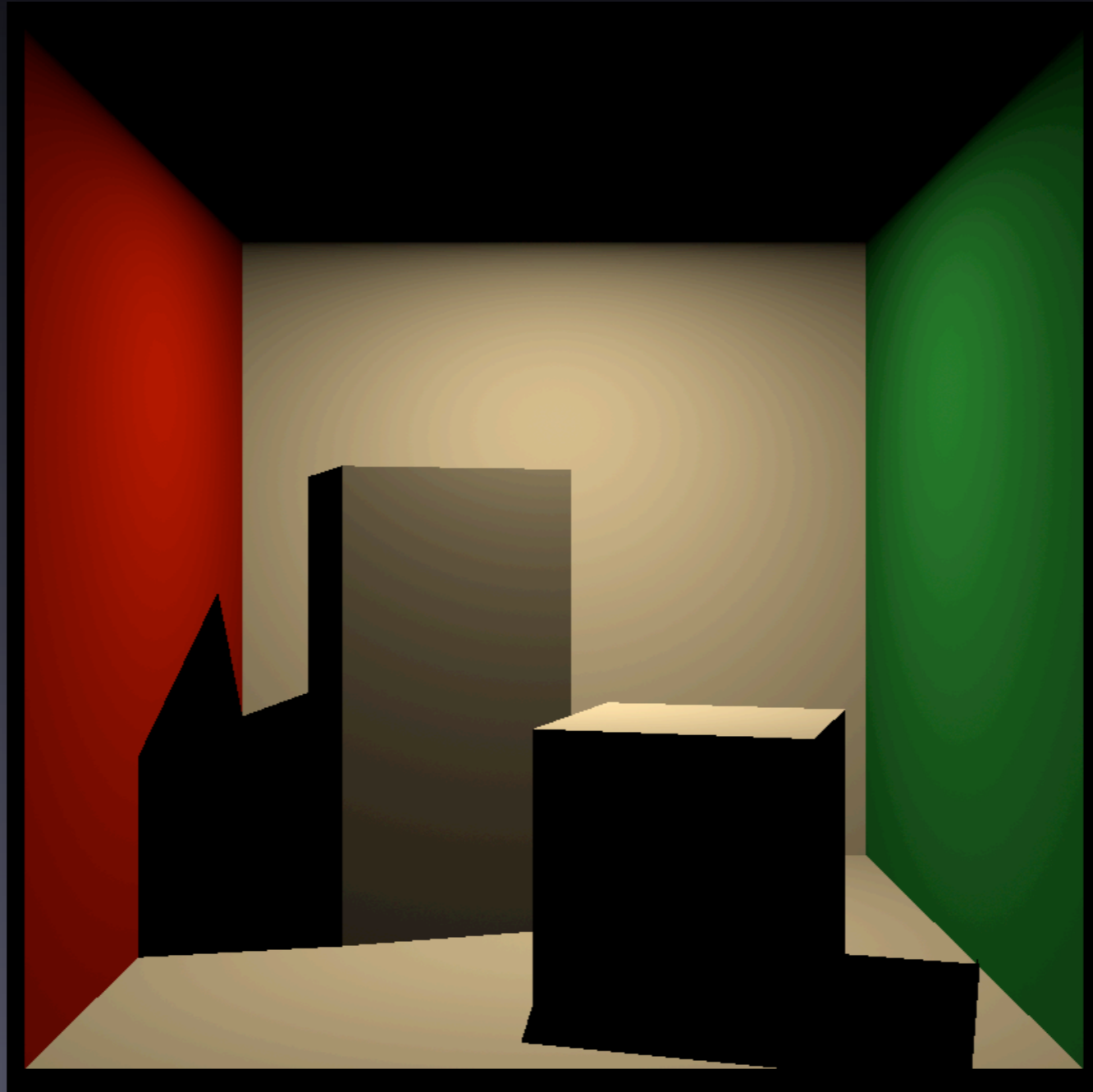
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- Extensions to Irradiance Caching

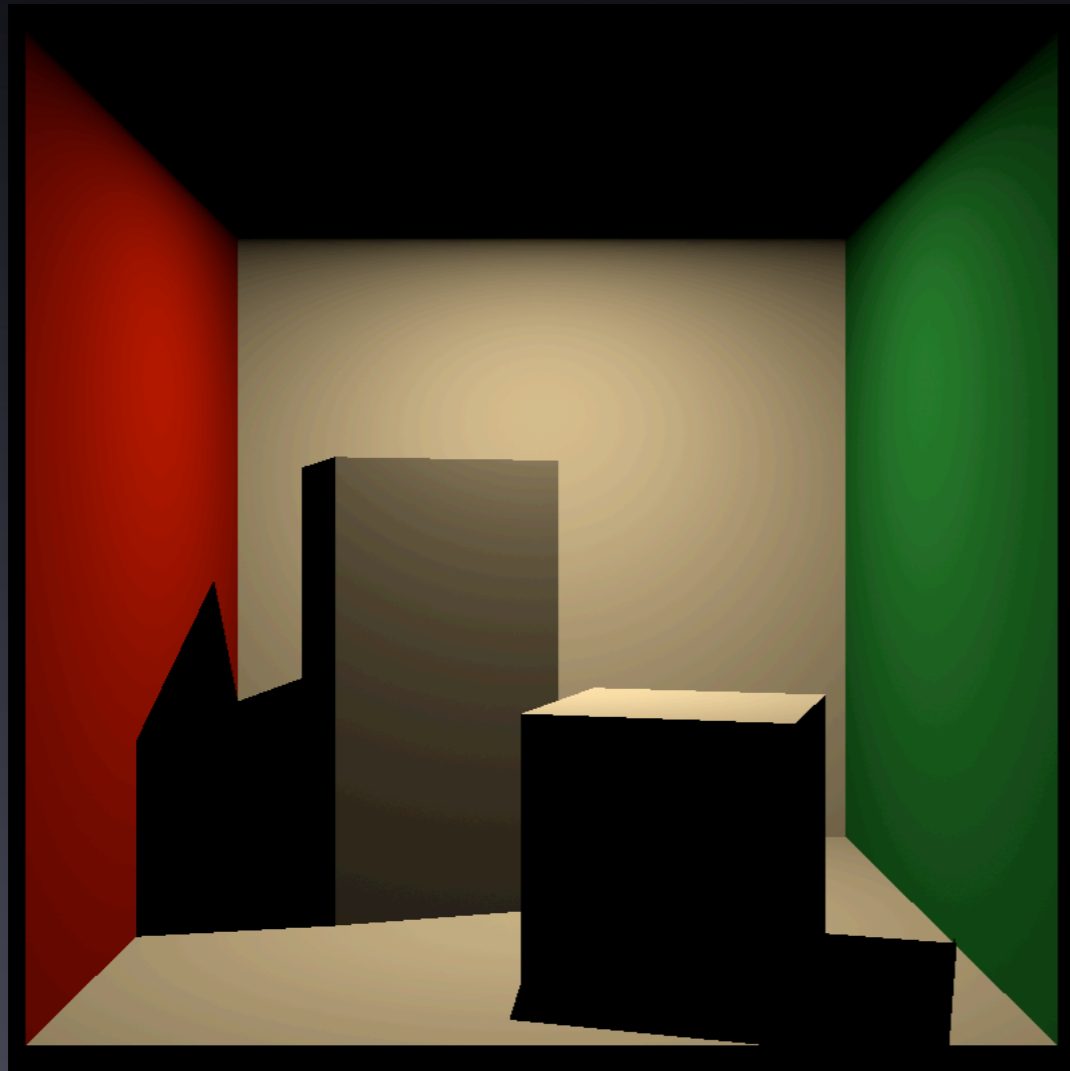
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- Extensions to Irradiance Caching
- Future Work

# Direct Illumination



# Comparison

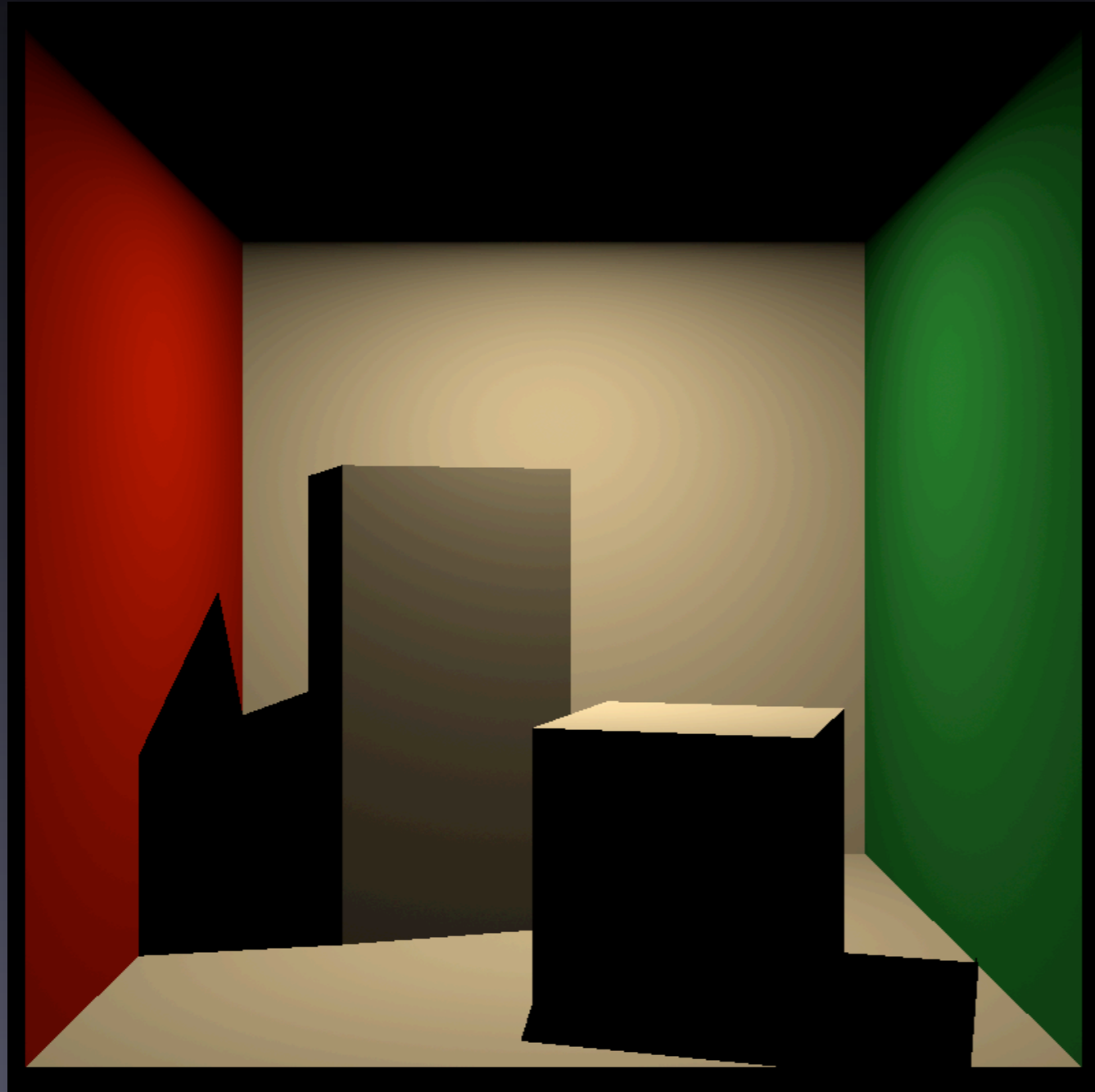


Simulated

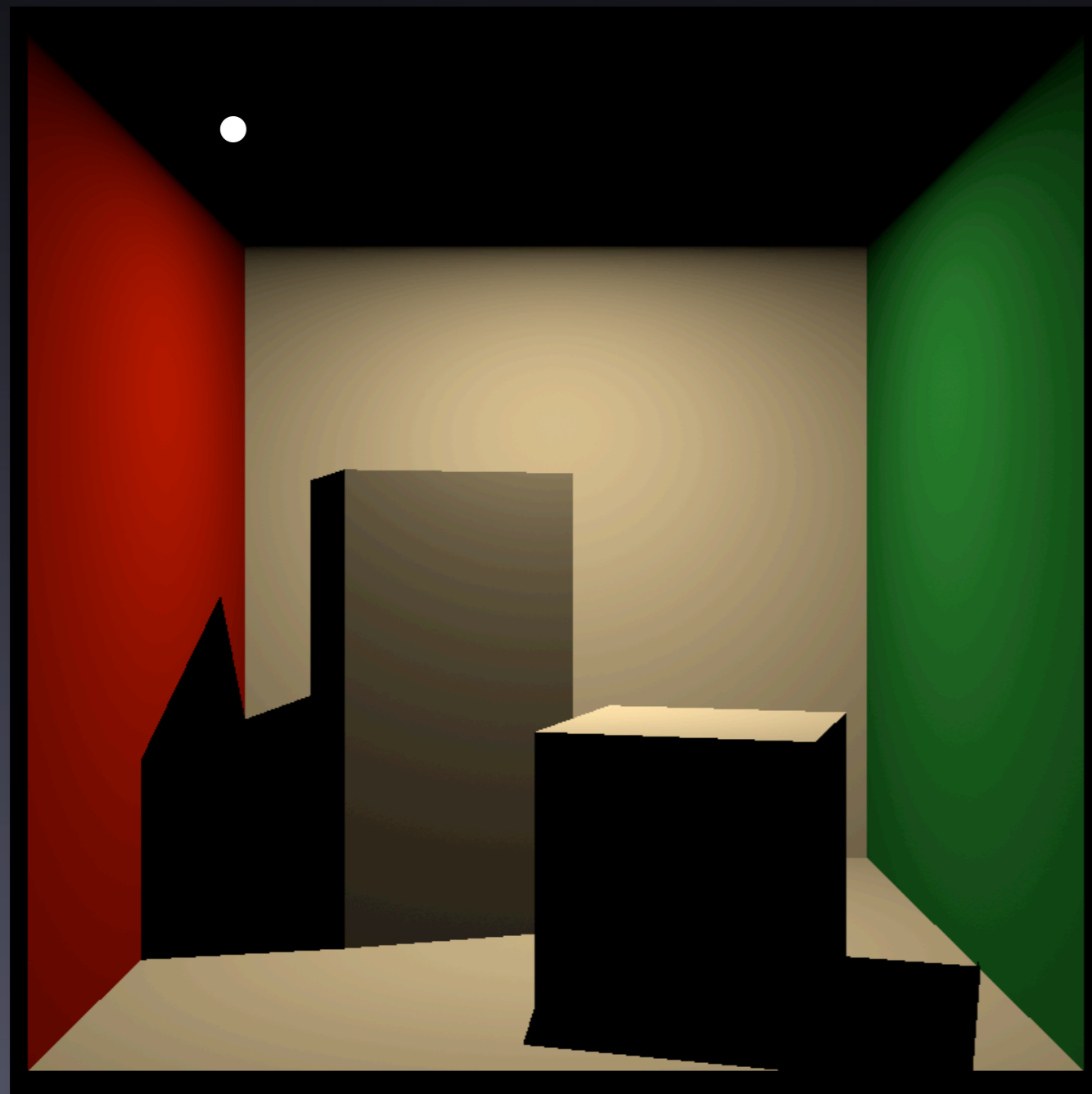


Real

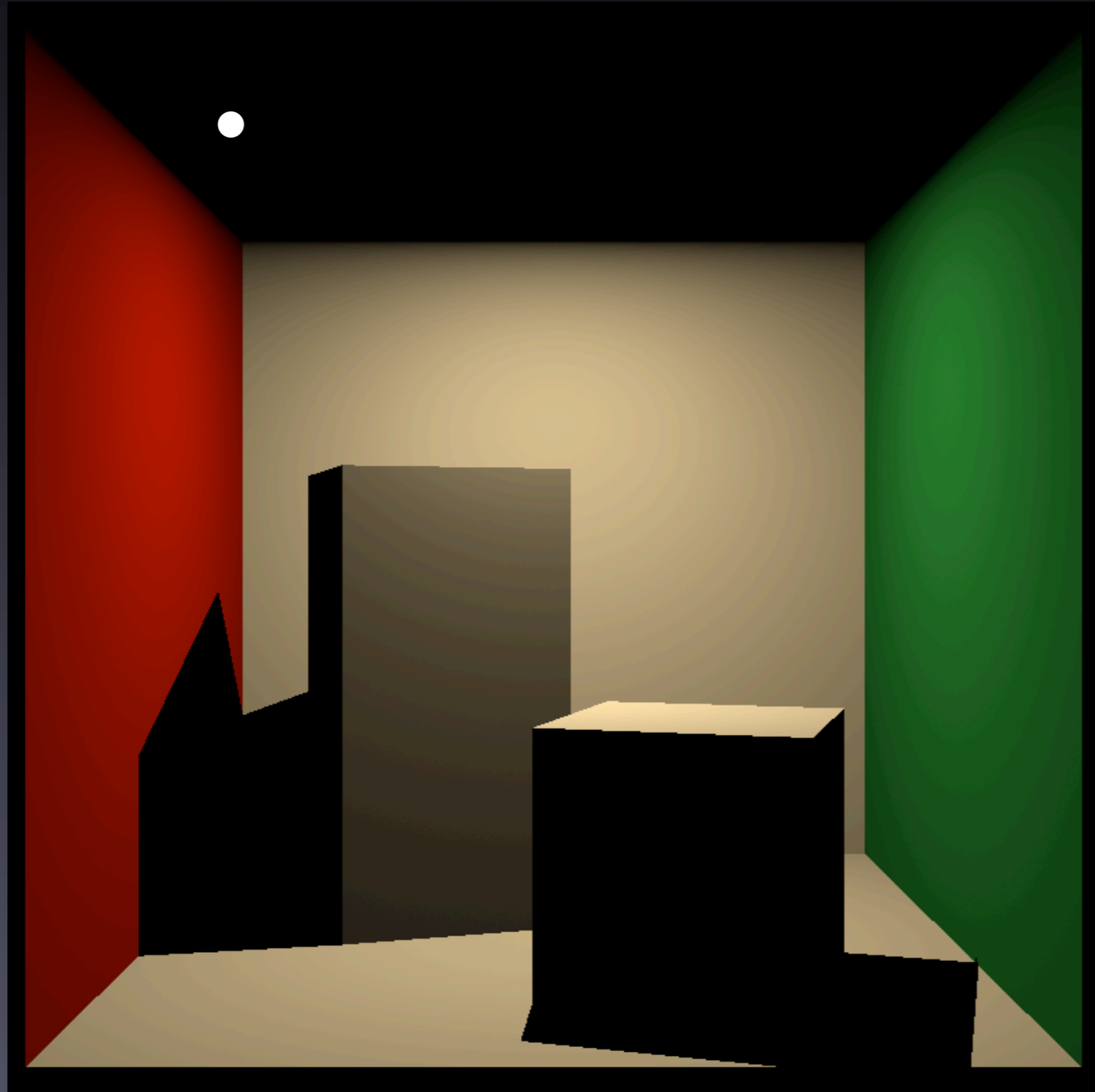
# Computing Indirect Illum.



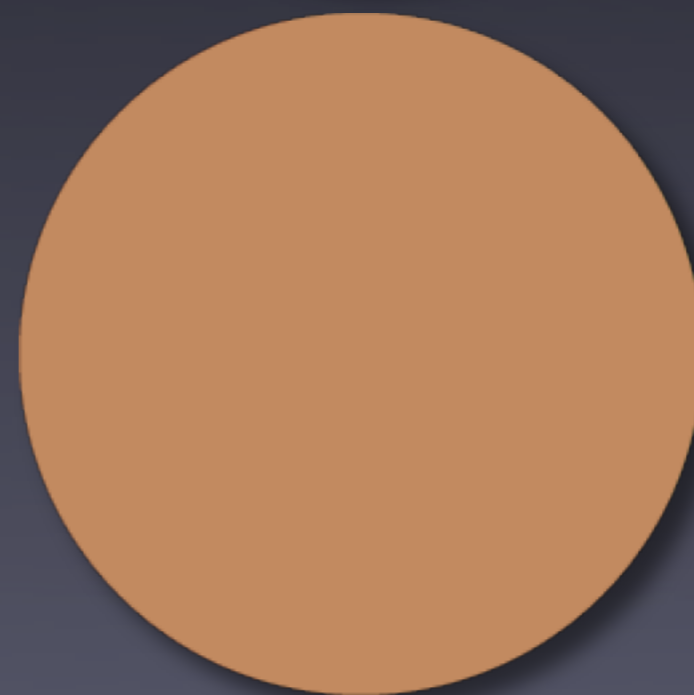
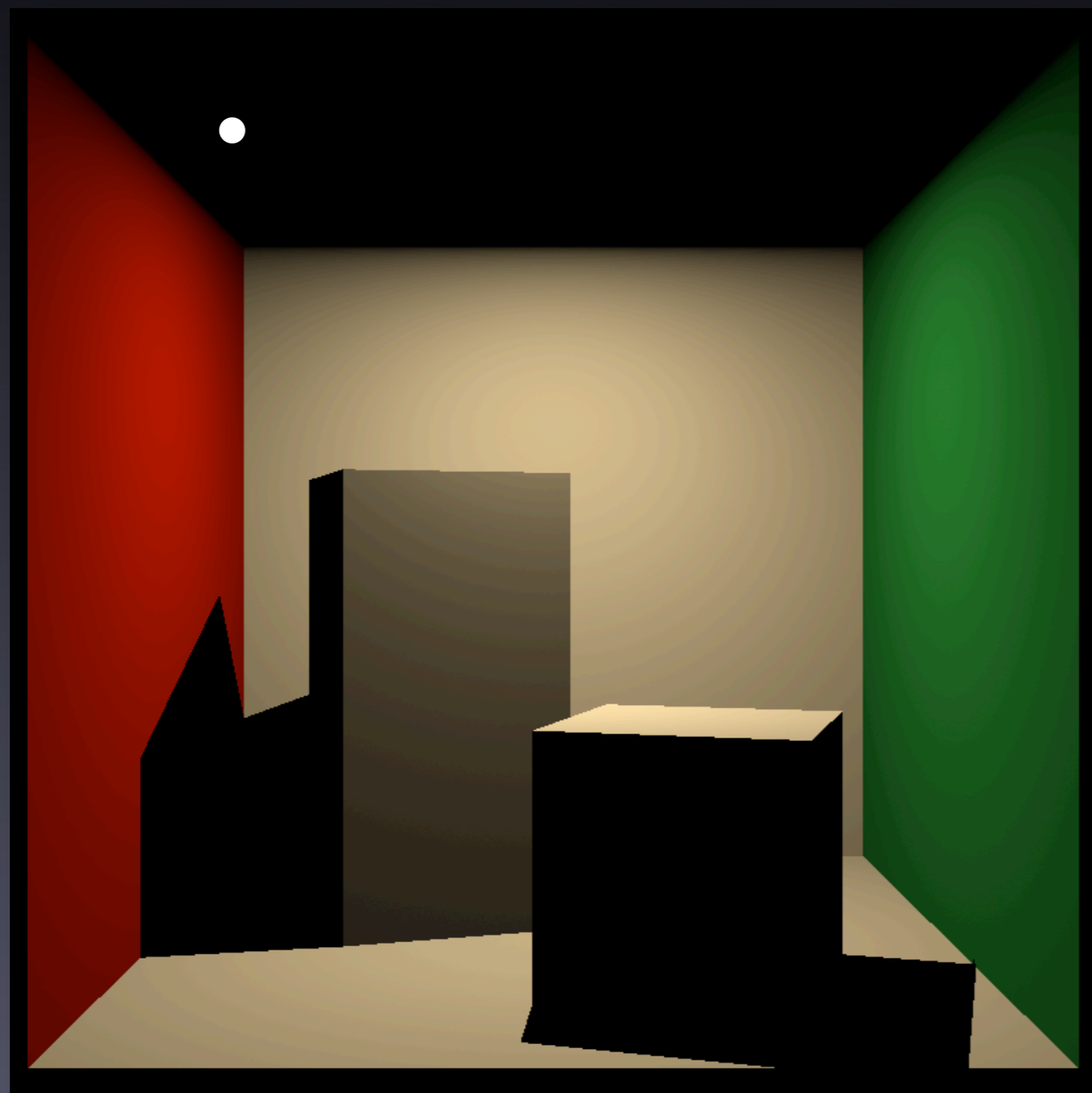
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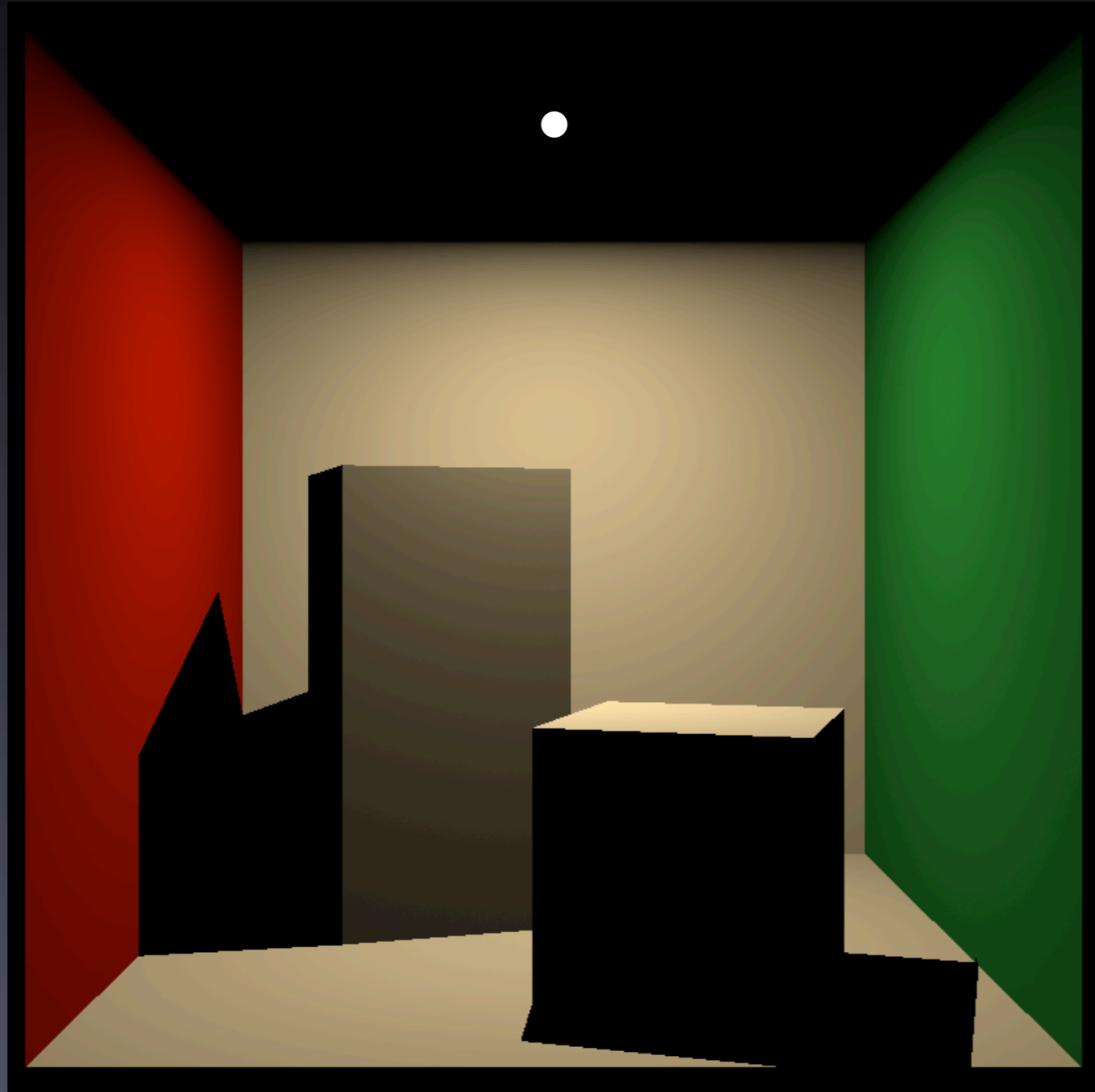
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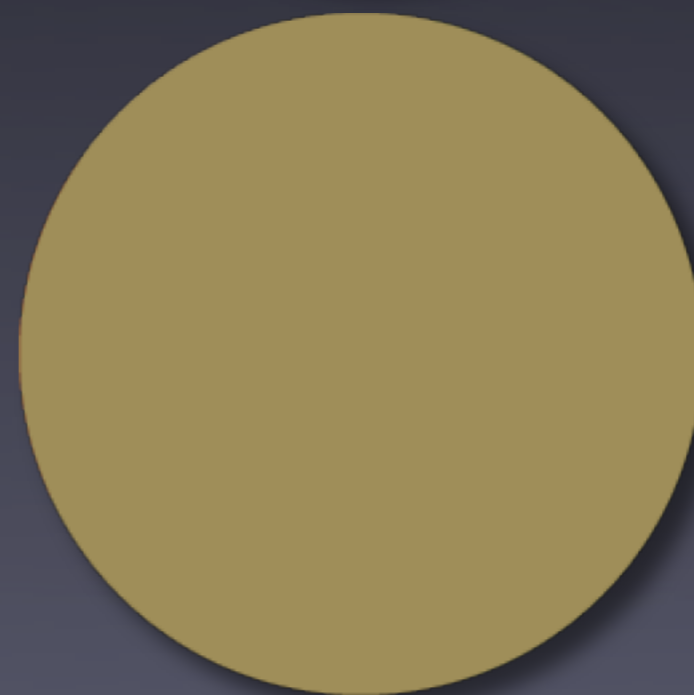
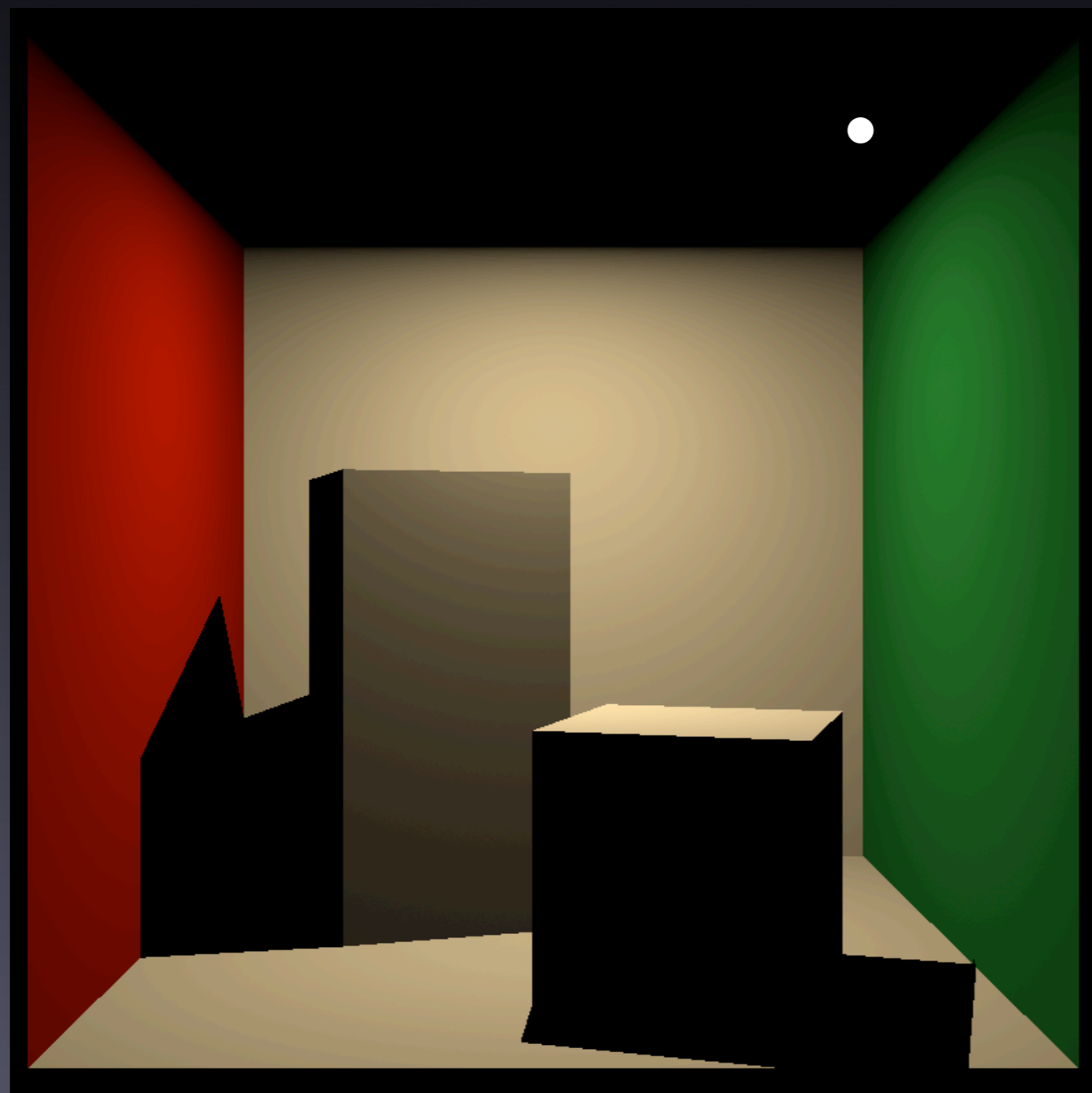
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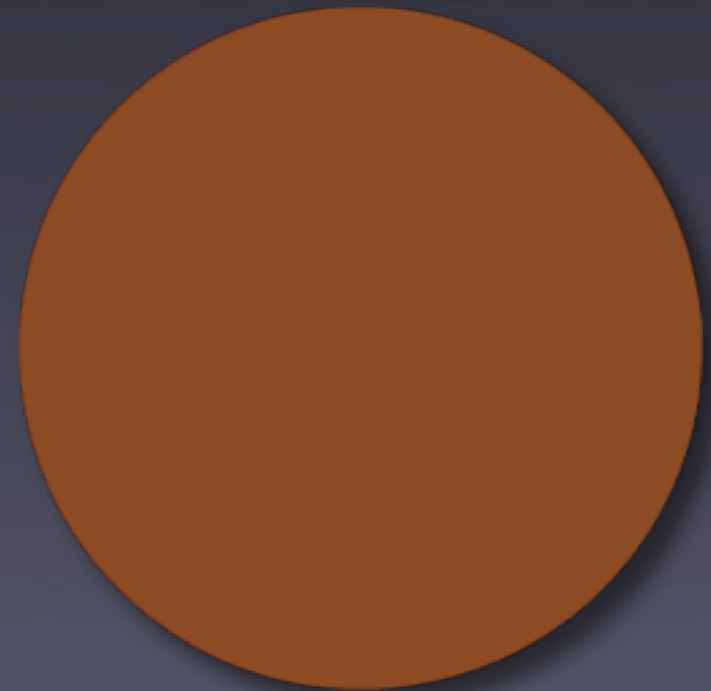
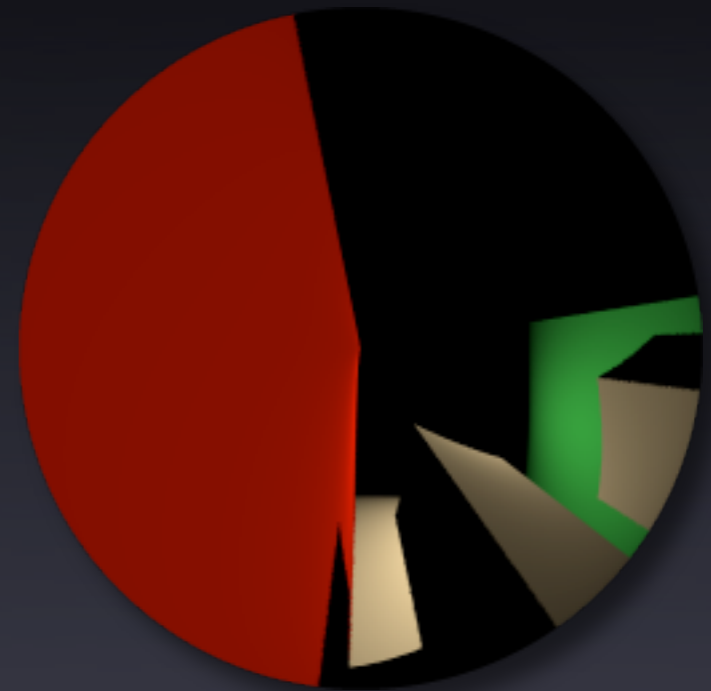
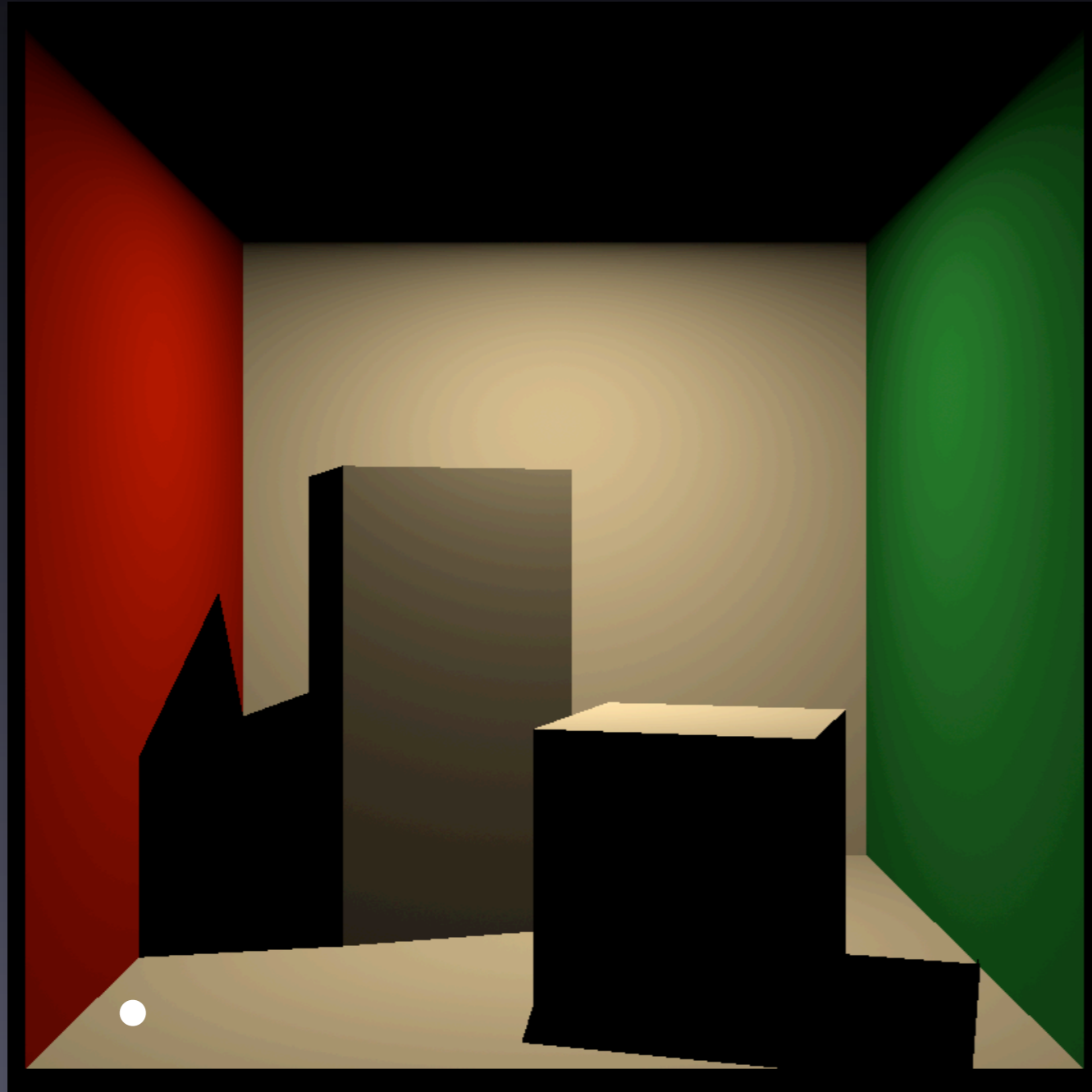
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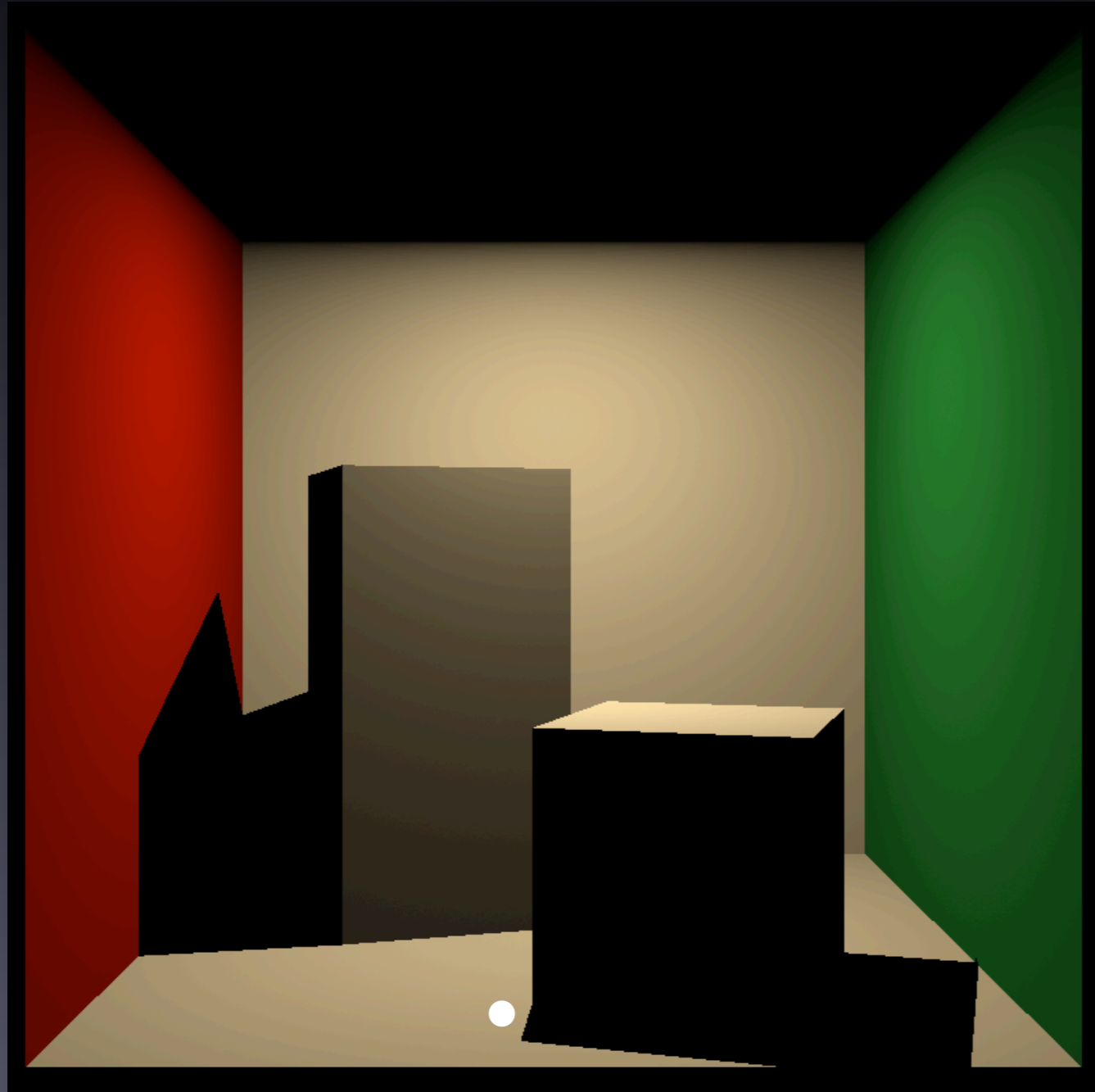
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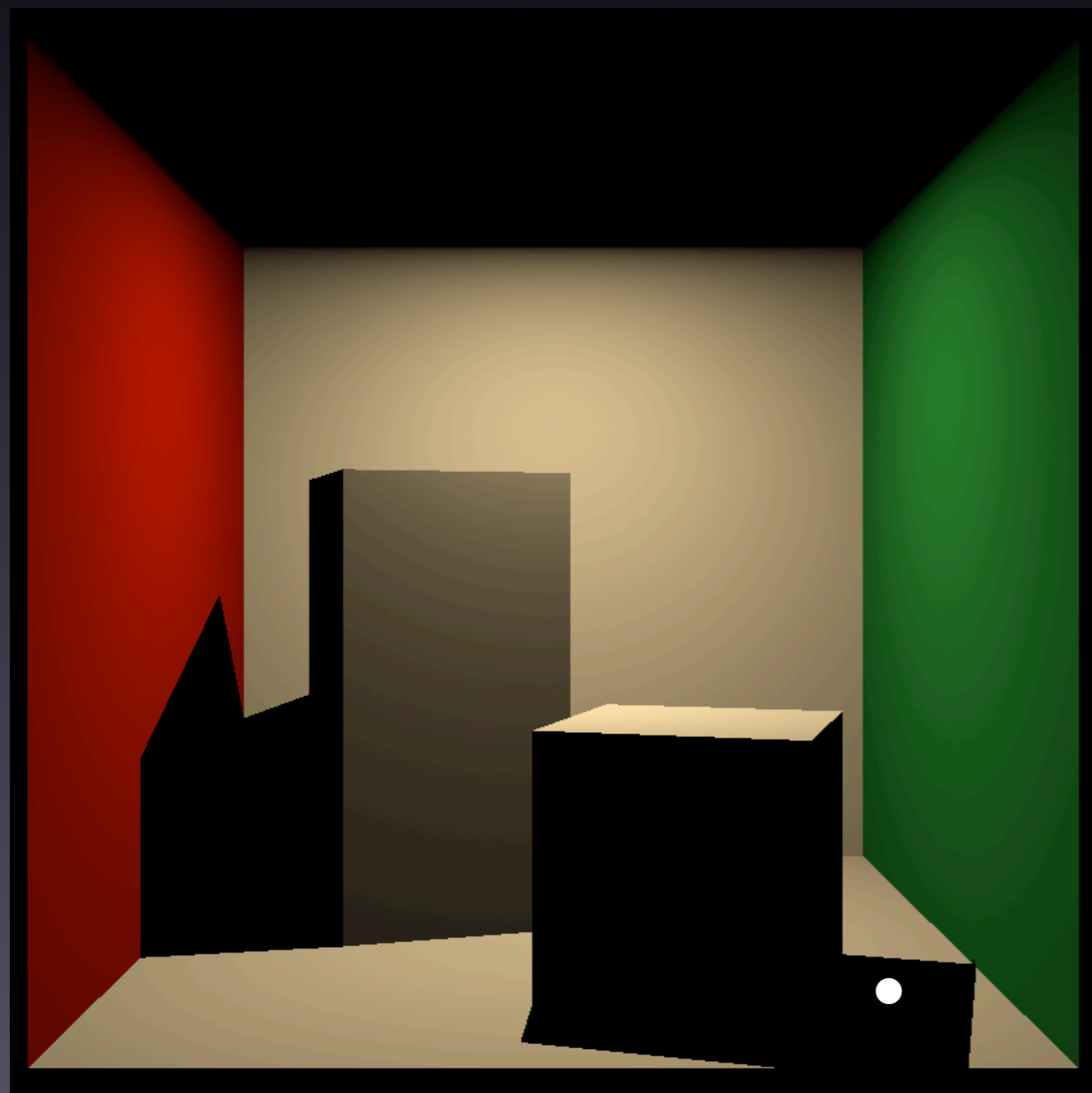
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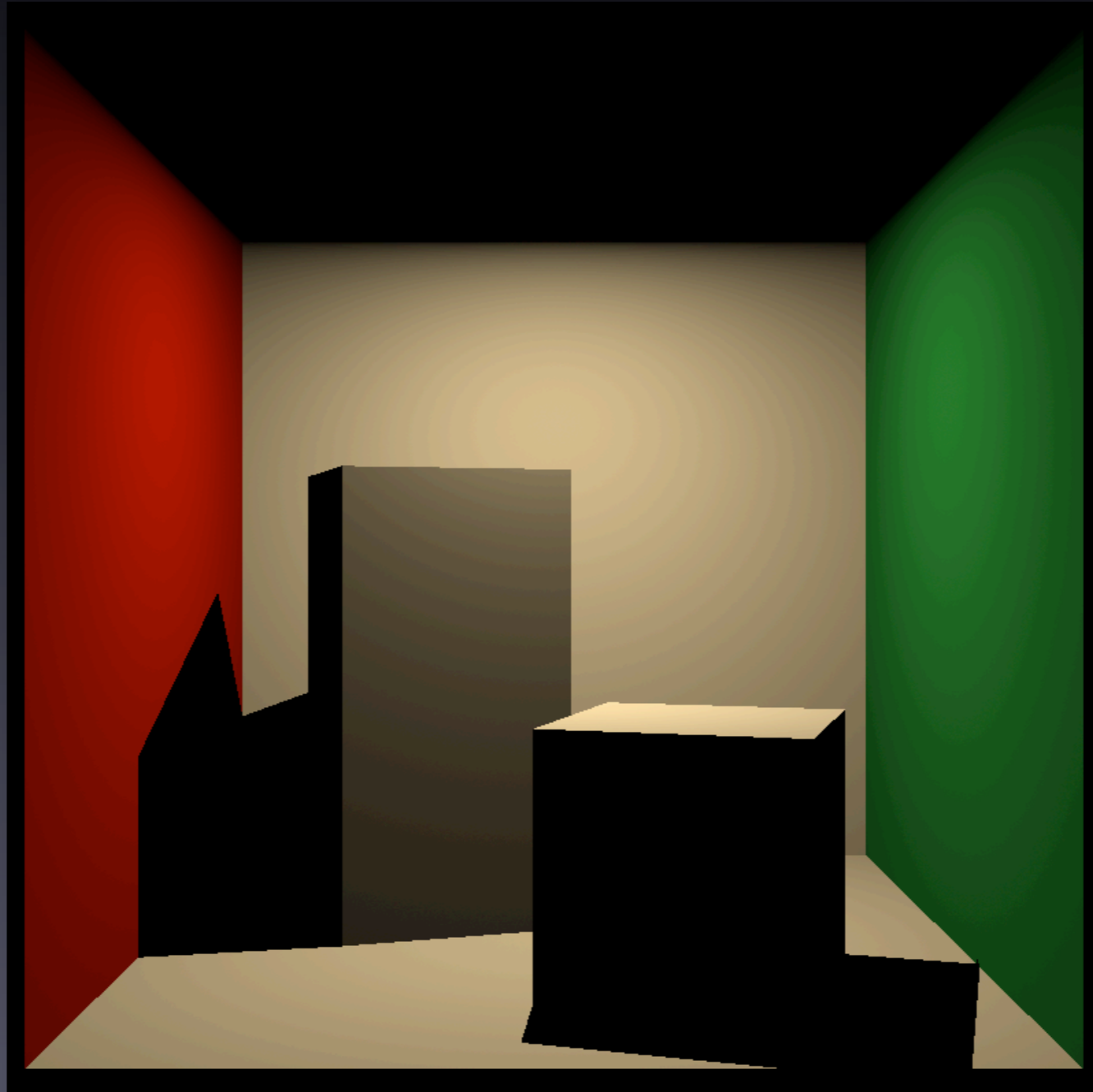
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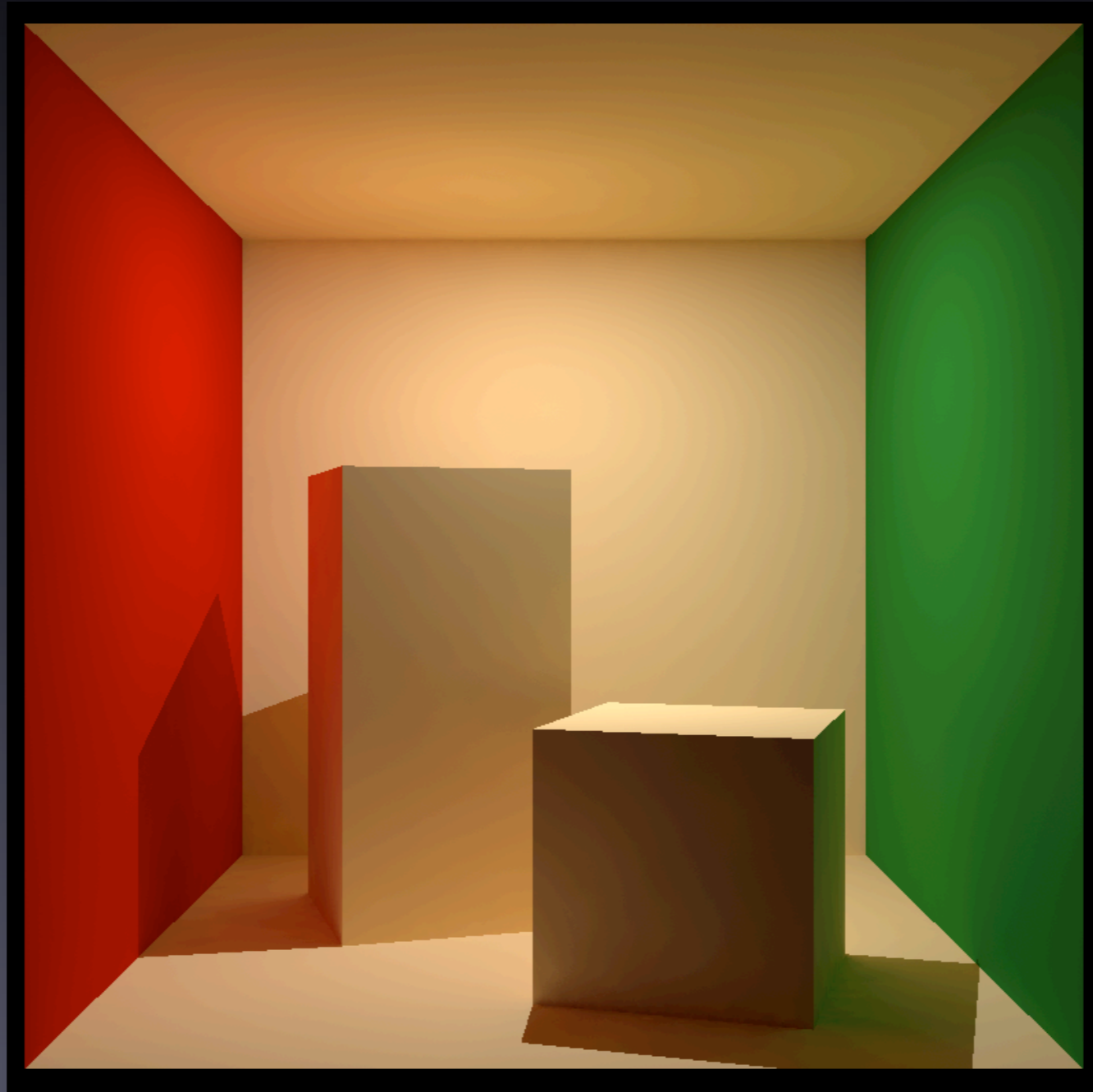
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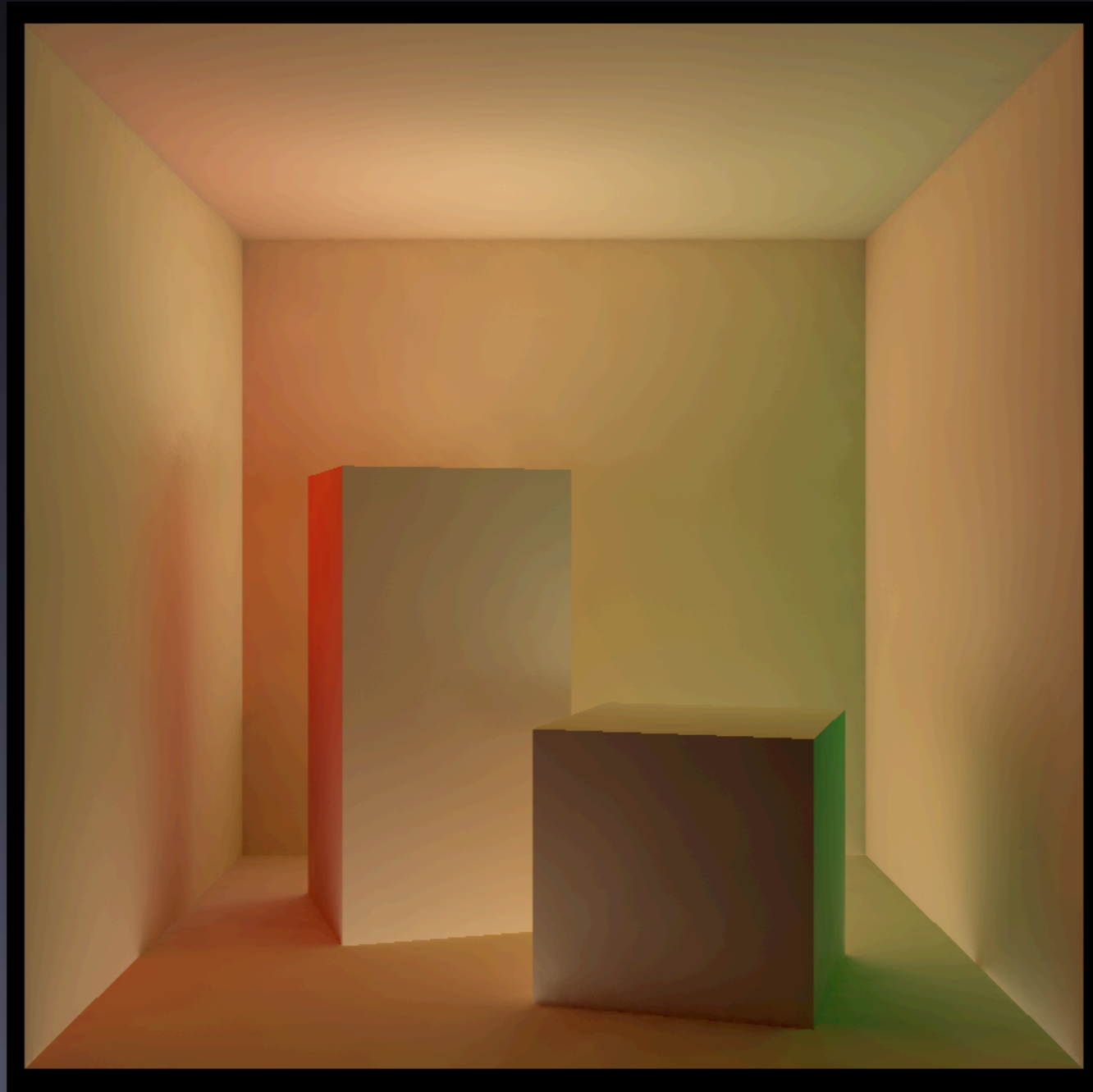
# Direct Illumination



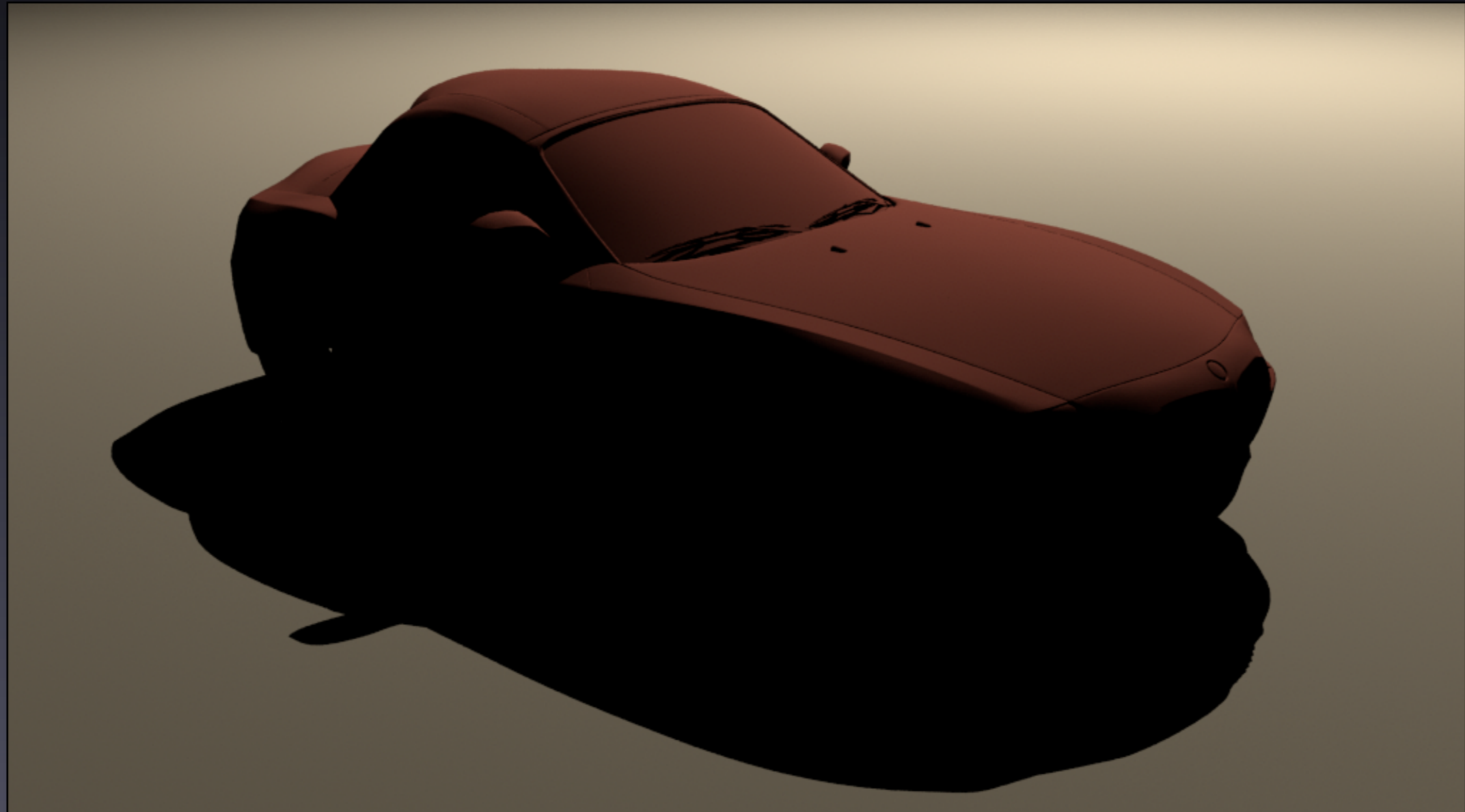
# Direct + Indirect Illum.



# Indirect Illumination



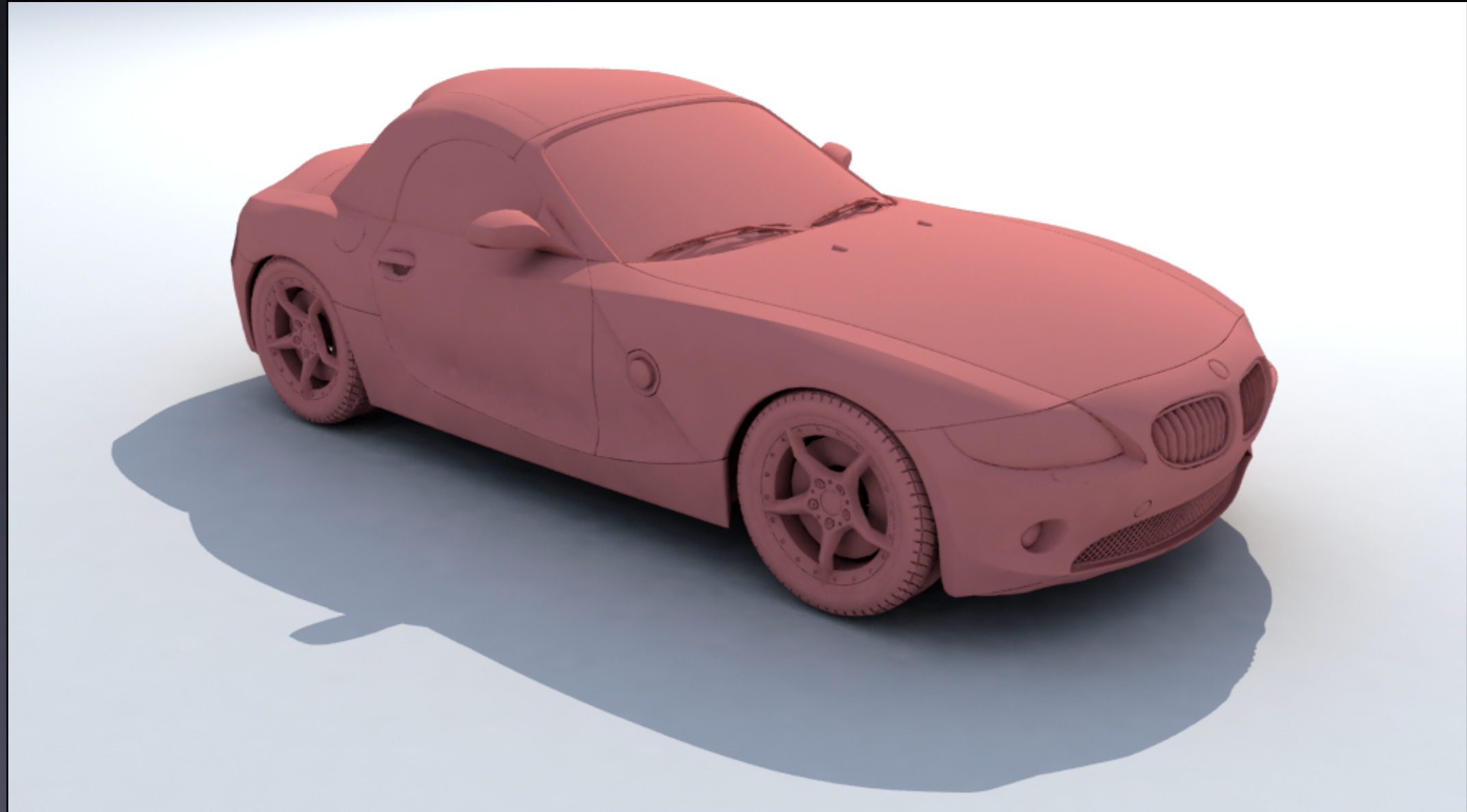
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# Direct Illumination



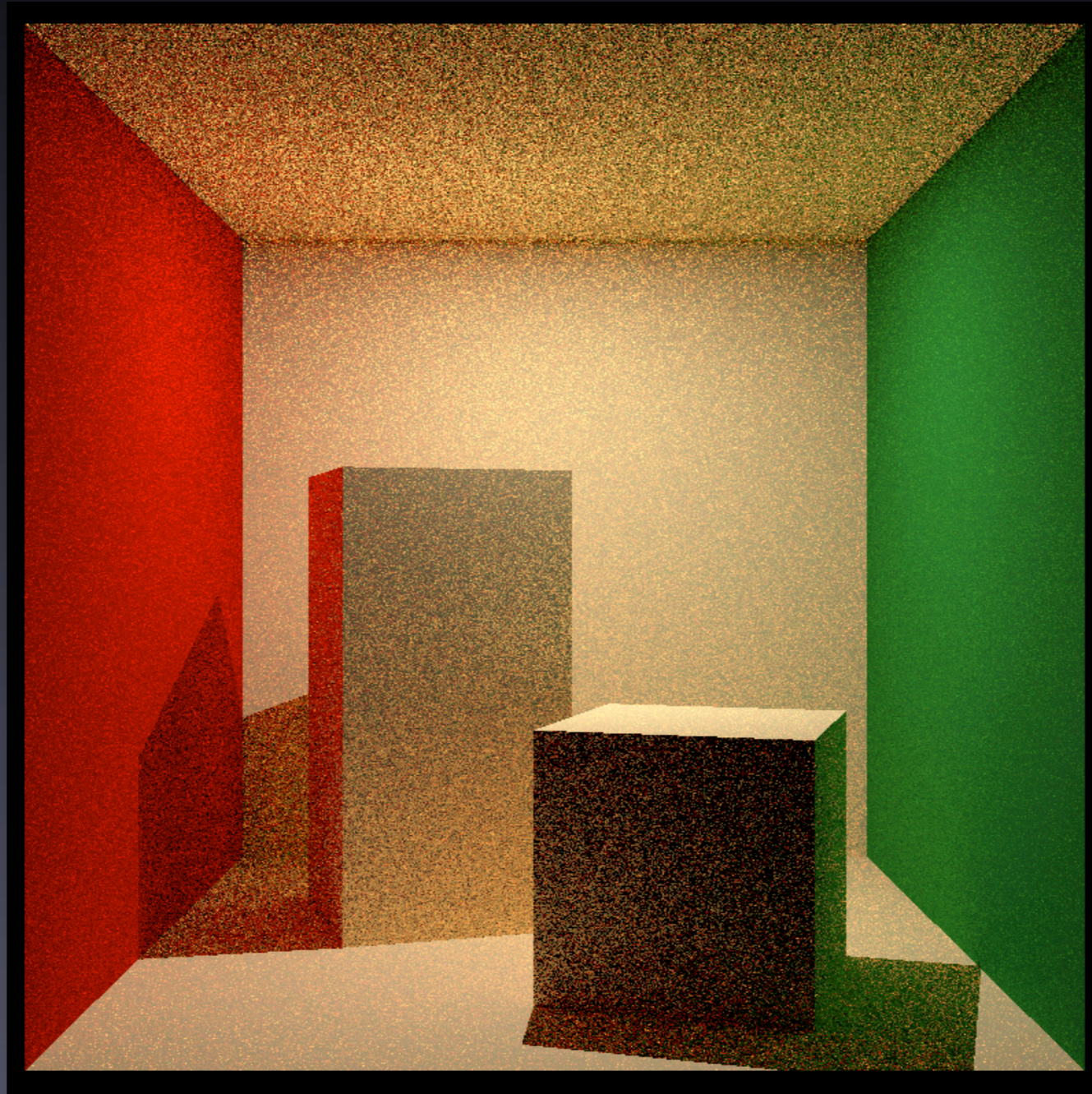
# Direct + Indirect Illum.



- Direct illumination is easy
- We will focus only on indirect illumination

# Path Tracing

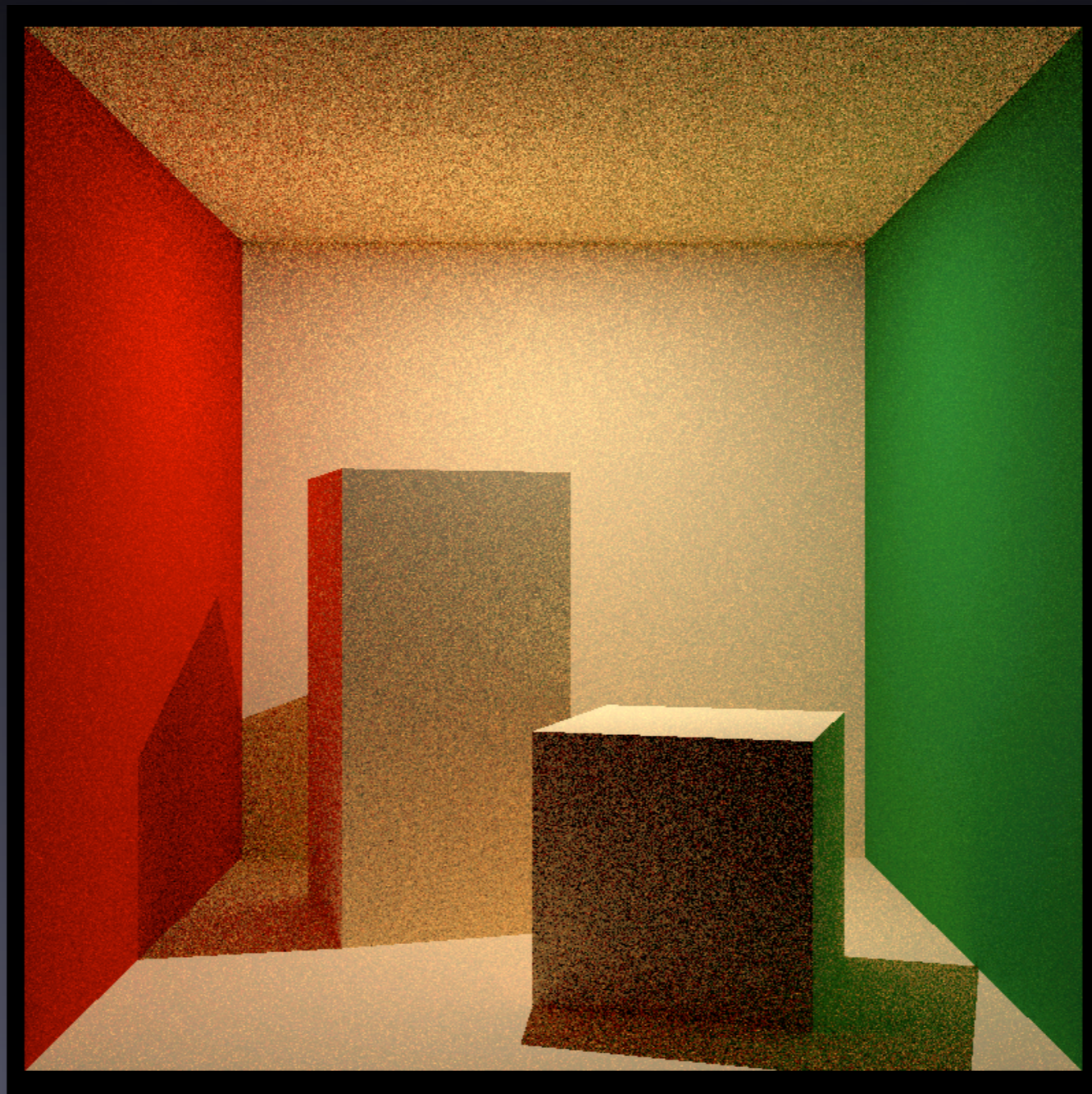
Kajiya '86



4 rays/pixel

# Path Tracing

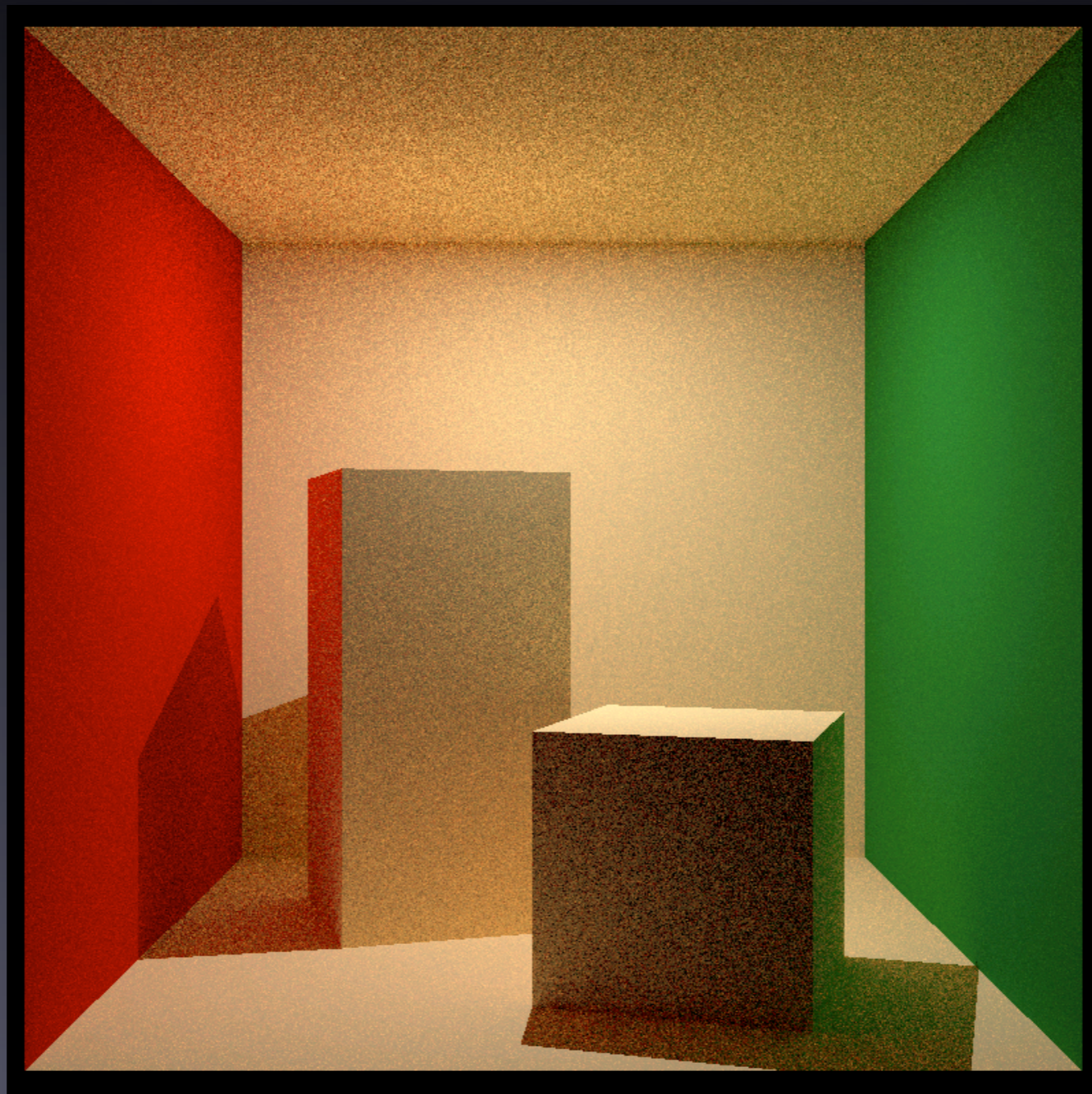
Kajiya '86



8 rays/pixel

# Path Tracing

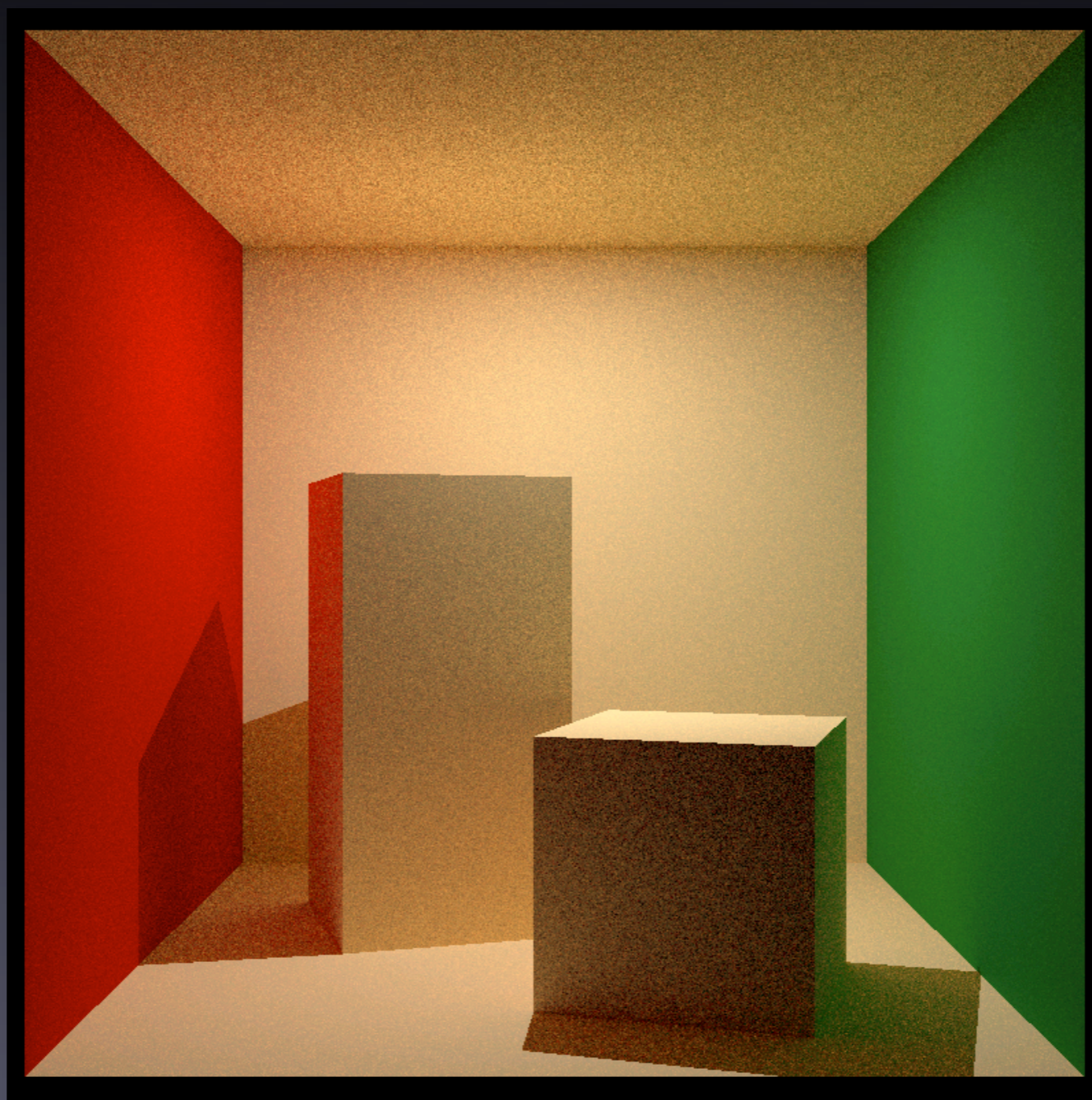
Kajiya '86



16 rays/pixel

# Path Tracing

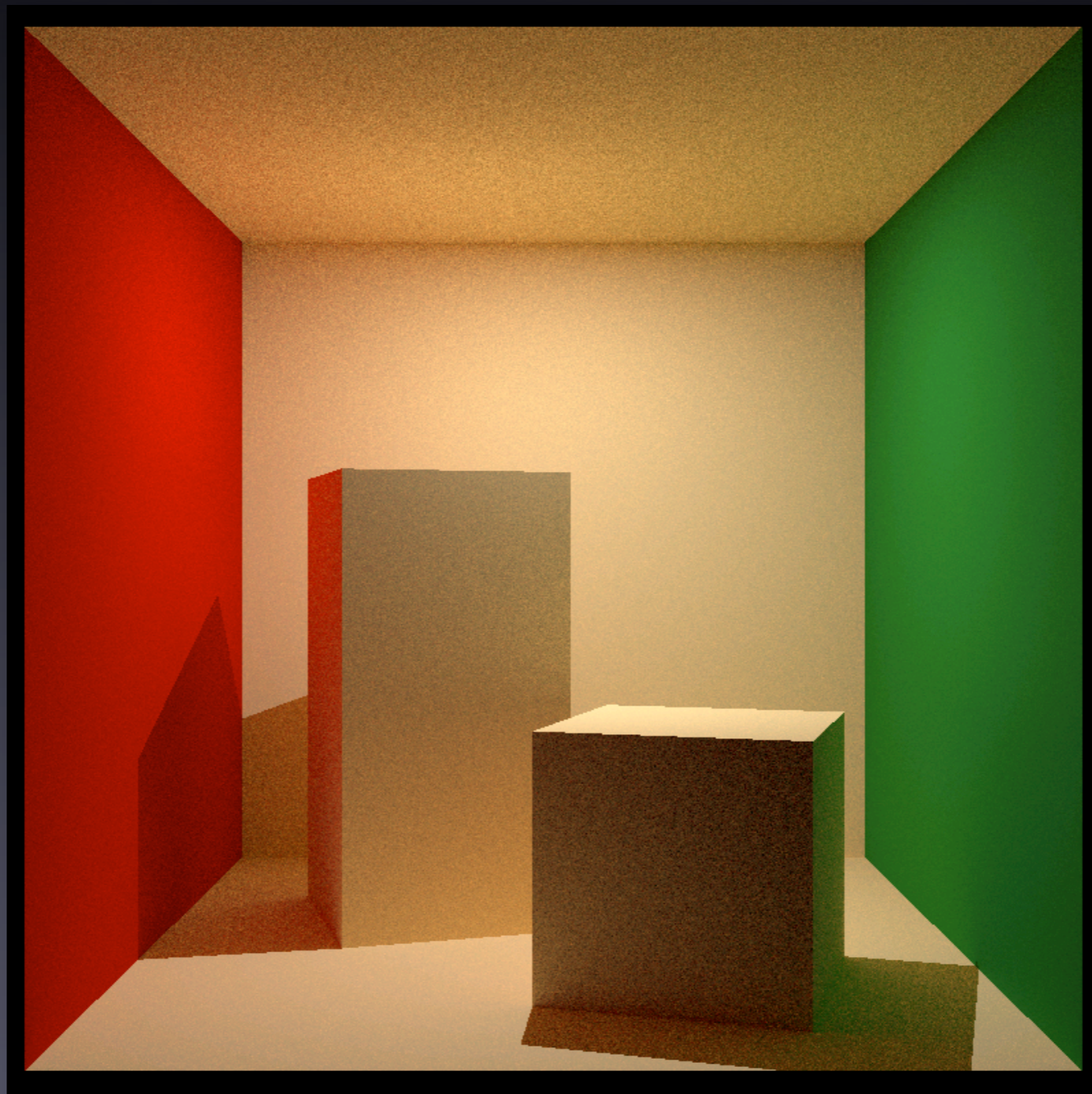
Kajiya '86



32 rays/pixel

# Path Tracing

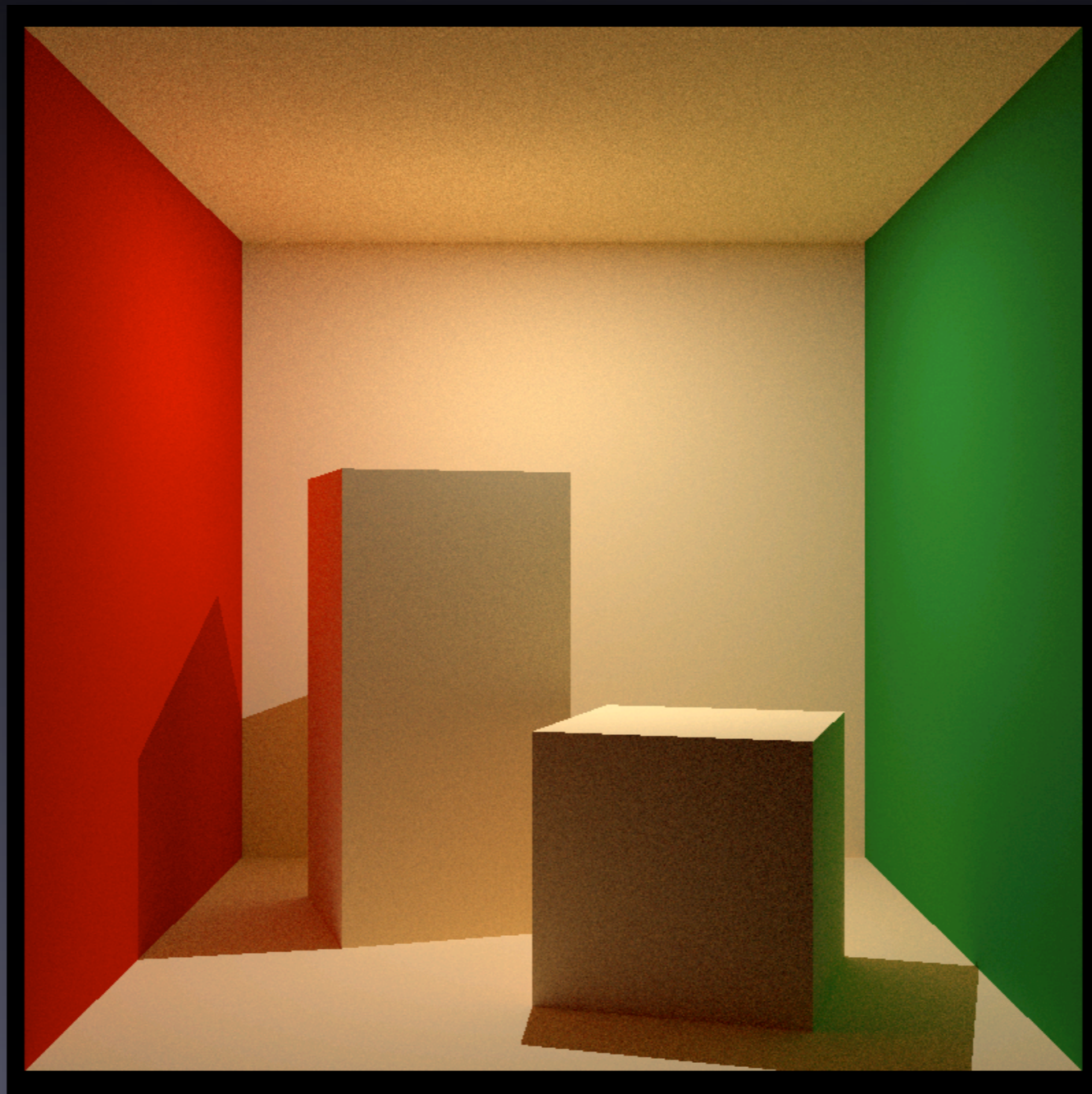
Kajiya '86



64 rays/pixel

# Path Tracing

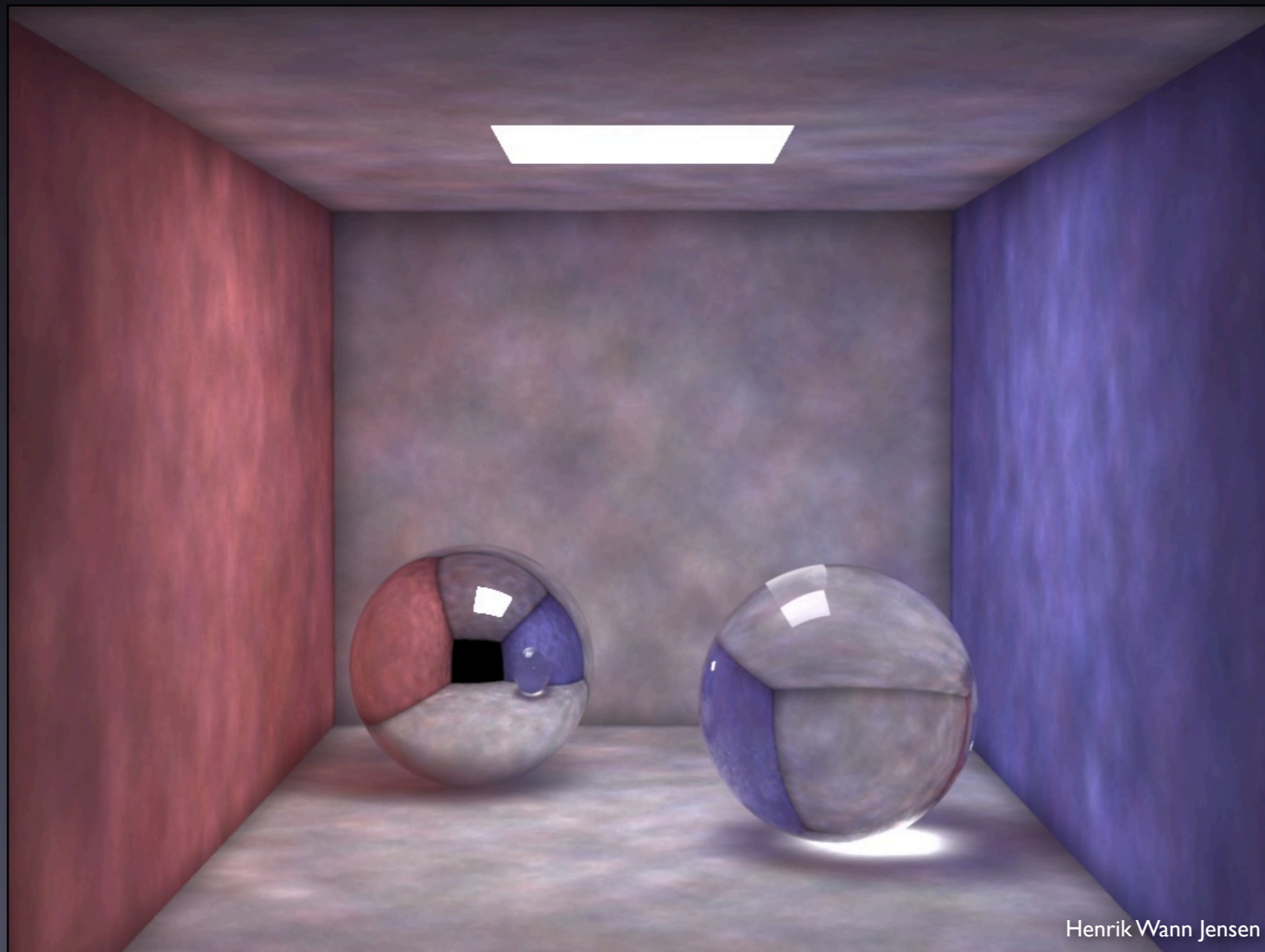
Kajiya '86



128 rays/pixel

# Photon Mapping

## Direct Visualization

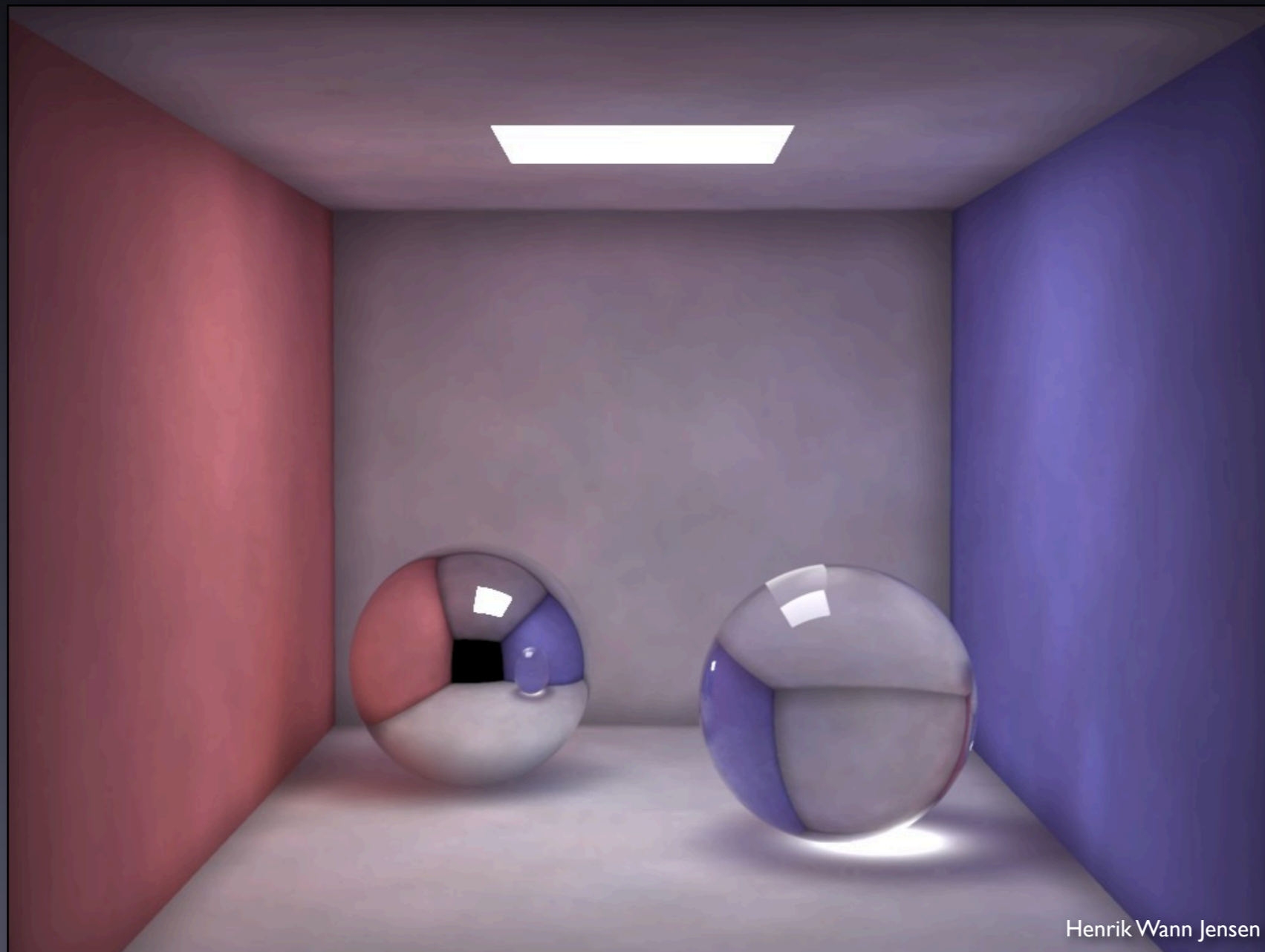


Henrik Wann Jensen

100000 photons / 50 photons in radiance estimate

# Photon Mapping

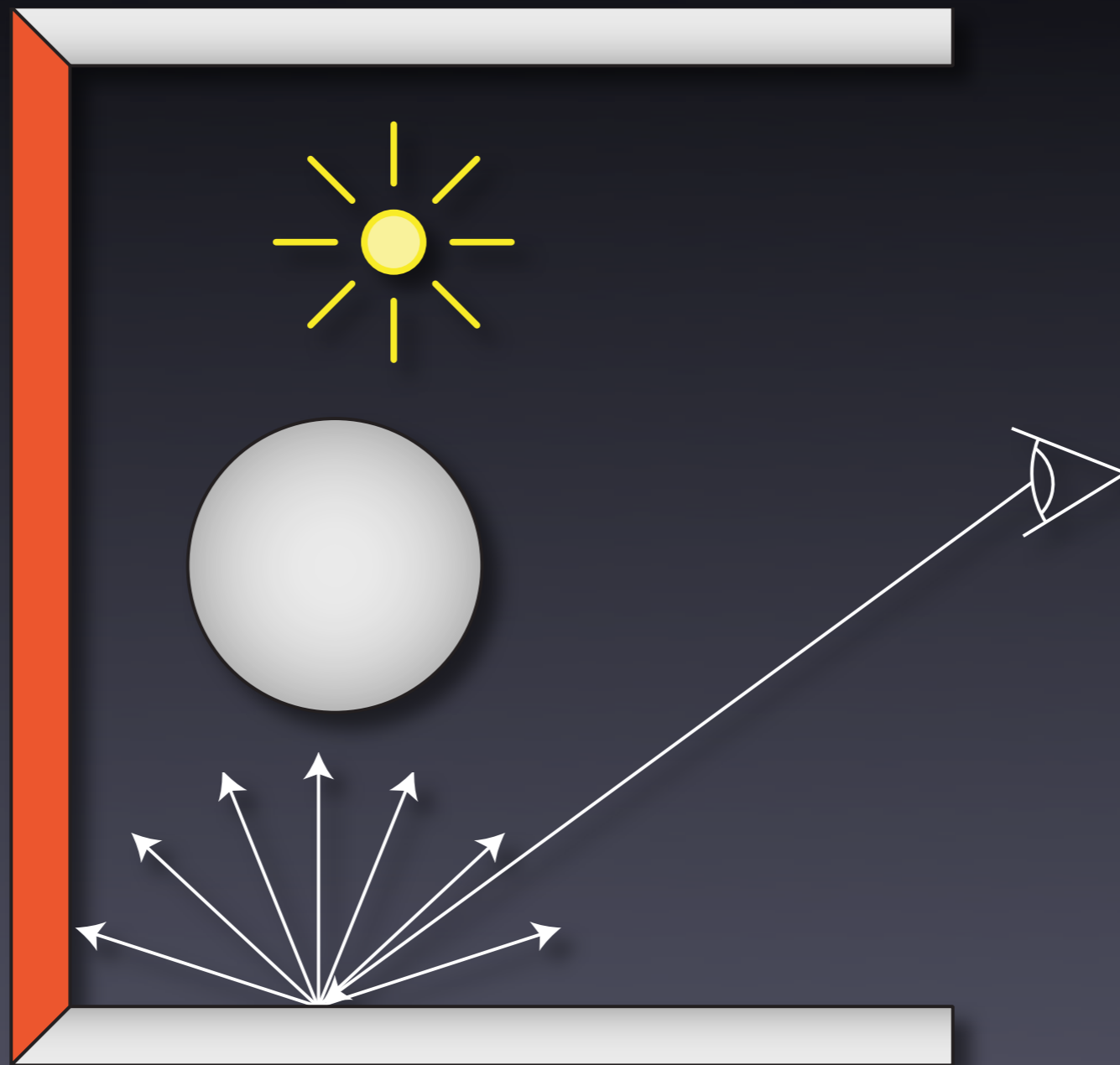
## Direct Visualization



Henrik Wann Jensen

500000 photons / 500 photons in radiance estimate

# Final Gather



# Motivation

- Tracing rays is costly.
- High-quality indirect illumination may require hundreds of gather rays per pixel for noise-free results.
- Hence, computing indirect illumination in this naive way is *very* costly.

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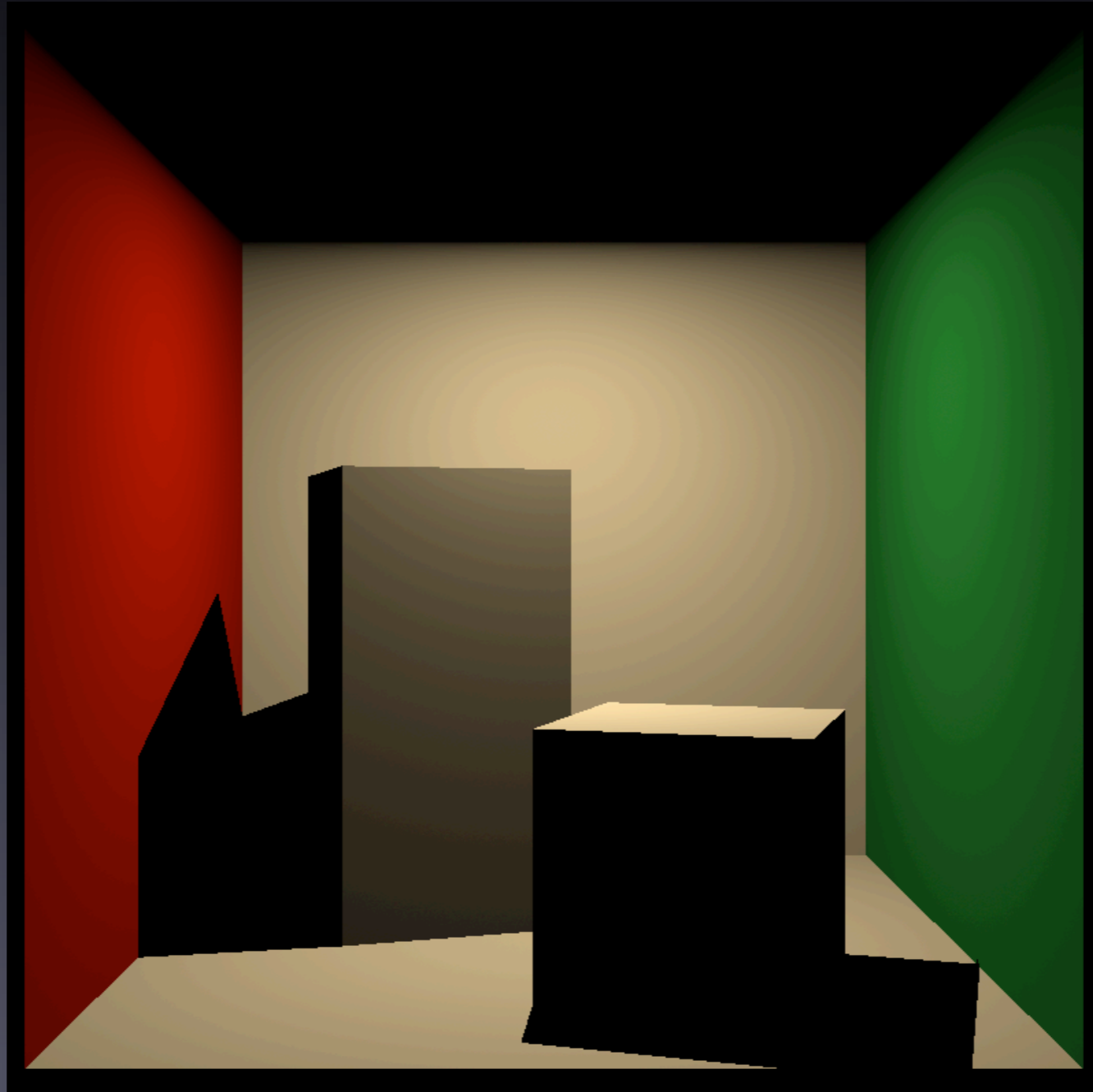
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# Irradiance Caching

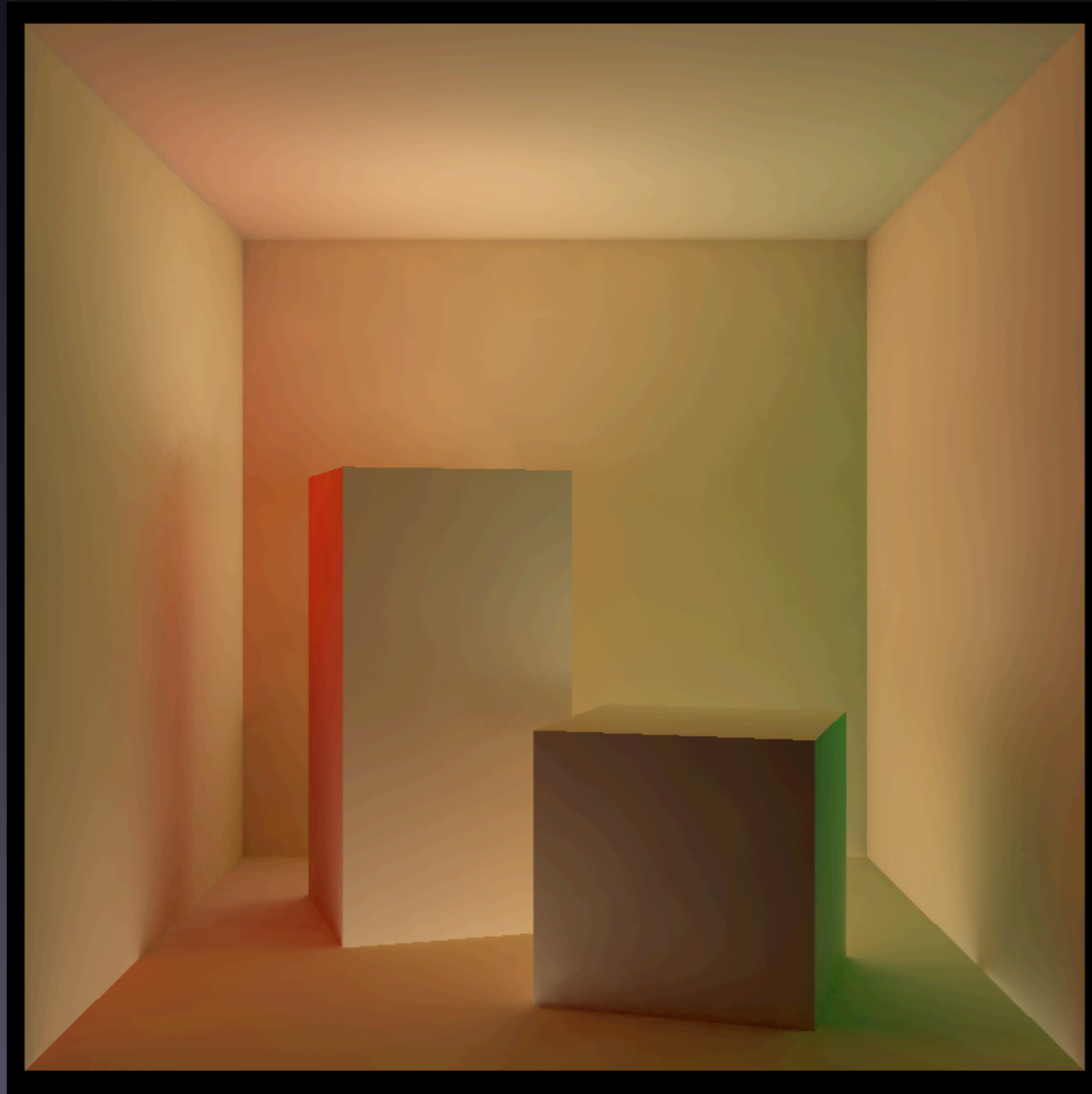
Ward et al. '88

- Irradiance caching was introduced by Ward et al. in 1988.

# Direct Illumination

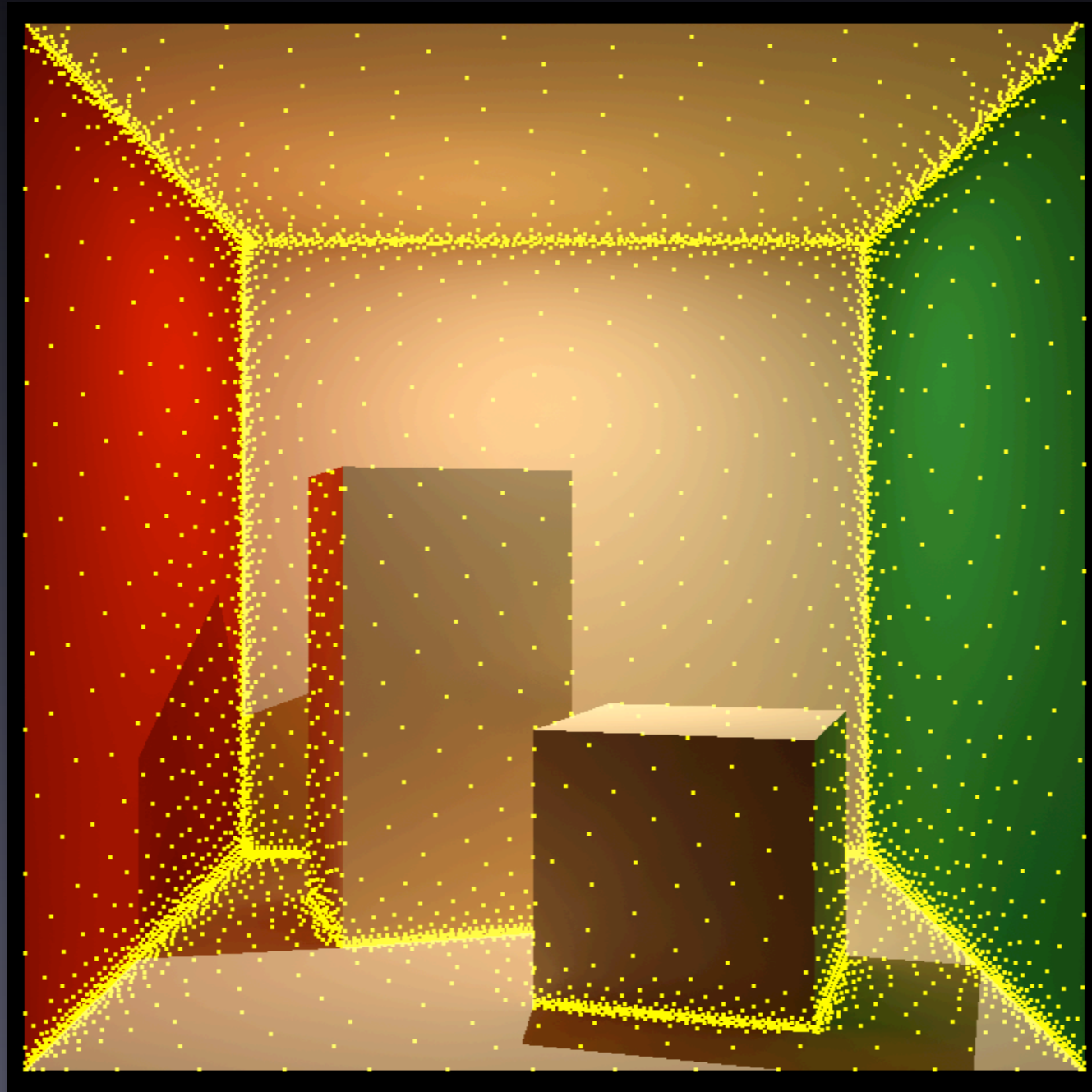


# Indirect Illumination



# Interpolate Indirect Illum.

Ward et al. '88



4621 samples

# Irradiance Caching

Ward et al. '88

- Some questions that remain:
  - How do we compute the indirect illumination values?
  - What criterion is used to determine whether a cache point is ``near?''
  - How do we interpolate the nearby cached values?

# What is Irradiance?

Ward et al. '88

$$E(\mathbf{x}) = \int_{\Omega} L_i(\mathbf{x}, \vec{\omega}_i) (\vec{\omega}_i \cdot \vec{n}) d\vec{\omega}_i$$

# What is Irradiance?

Ward et al. '88

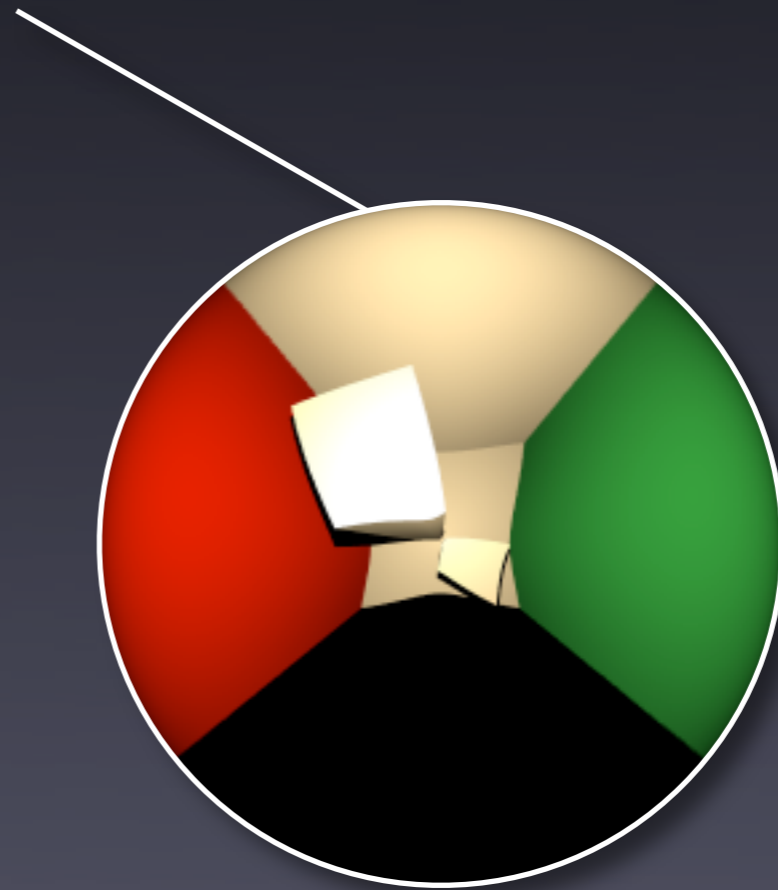
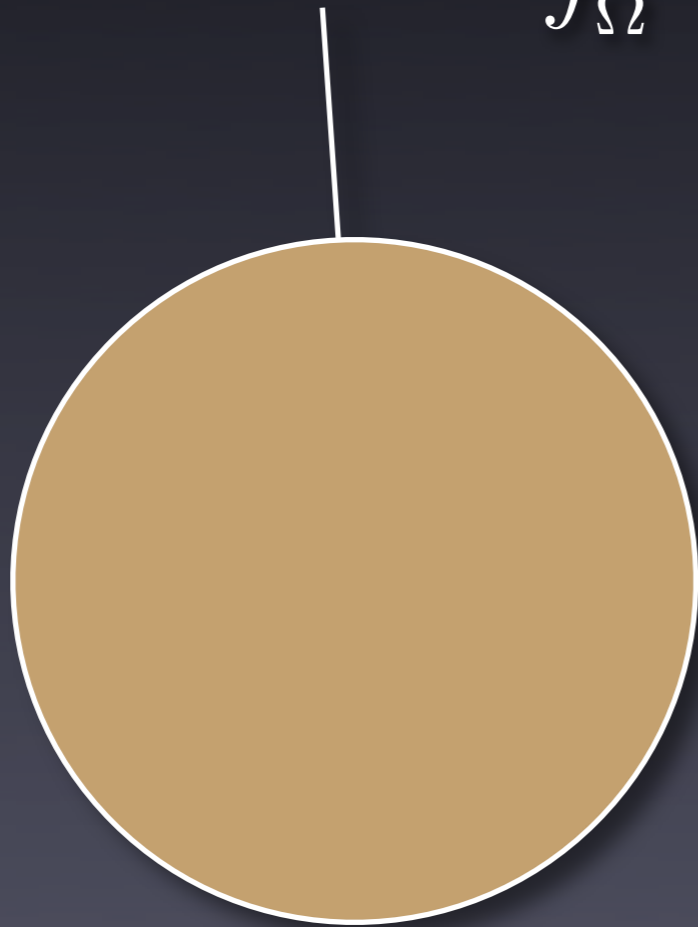
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# What is Irradiance?

Ward et al. '88

$$E(\mathbf{x}) = \int_{\Omega} L_i(\mathbf{x}, \vec{\omega}_i) (\vec{\omega}_i \cdot \vec{n}) d\vec{\omega}_i$$



# What is Irradiance?

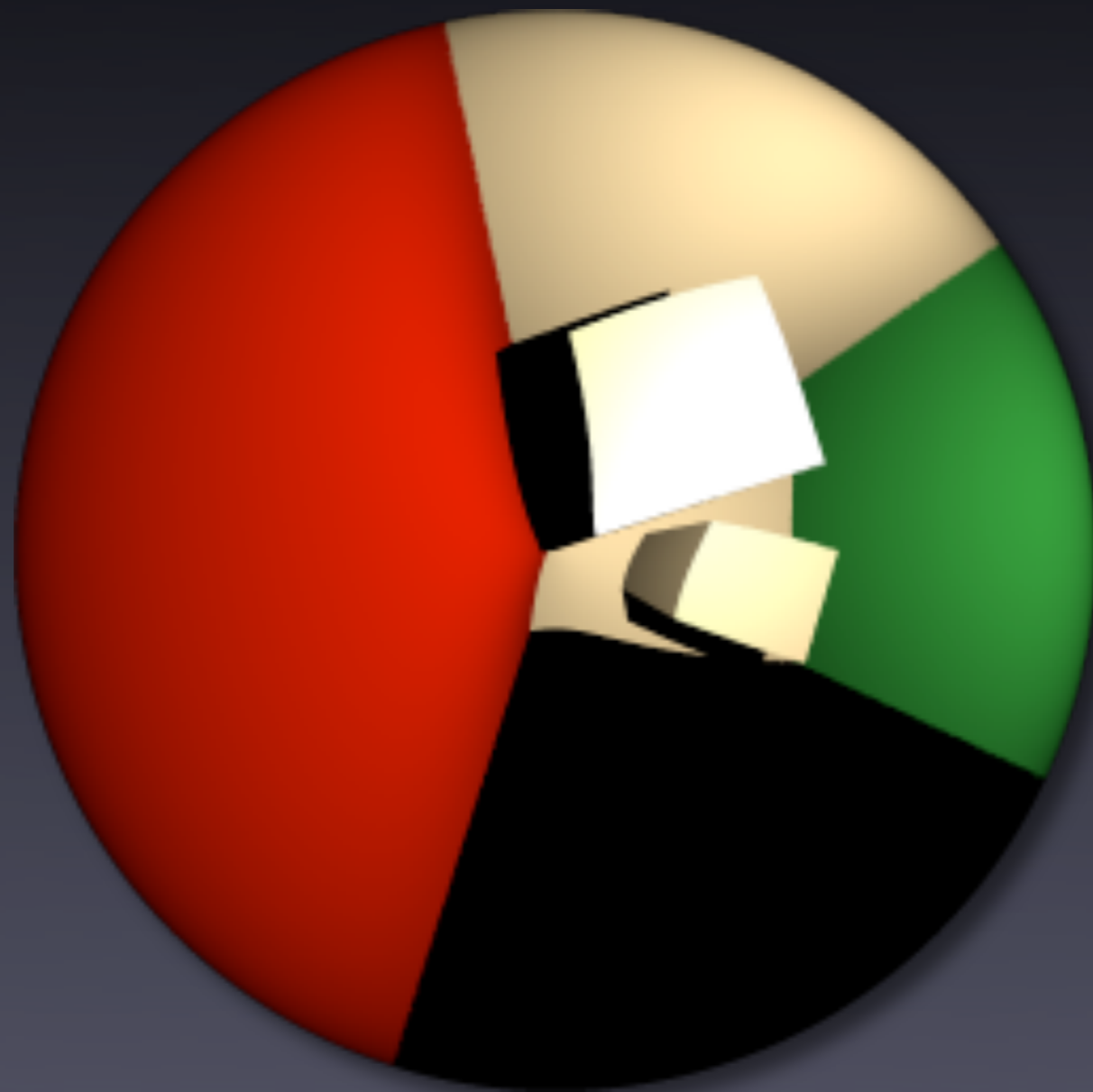
Ward et al. '88

$$E(\mathbf{x}) = \int_{\Omega} L_i(\mathbf{x}, \vec{\omega}_i) (\vec{\omega}_i \cdot \vec{n}) d\vec{\omega}_i$$

Nearly impossible to solve analytically!

# Computing Irradiance

Ward et al. '88



# Computing Irradiance

Ward et al. '88



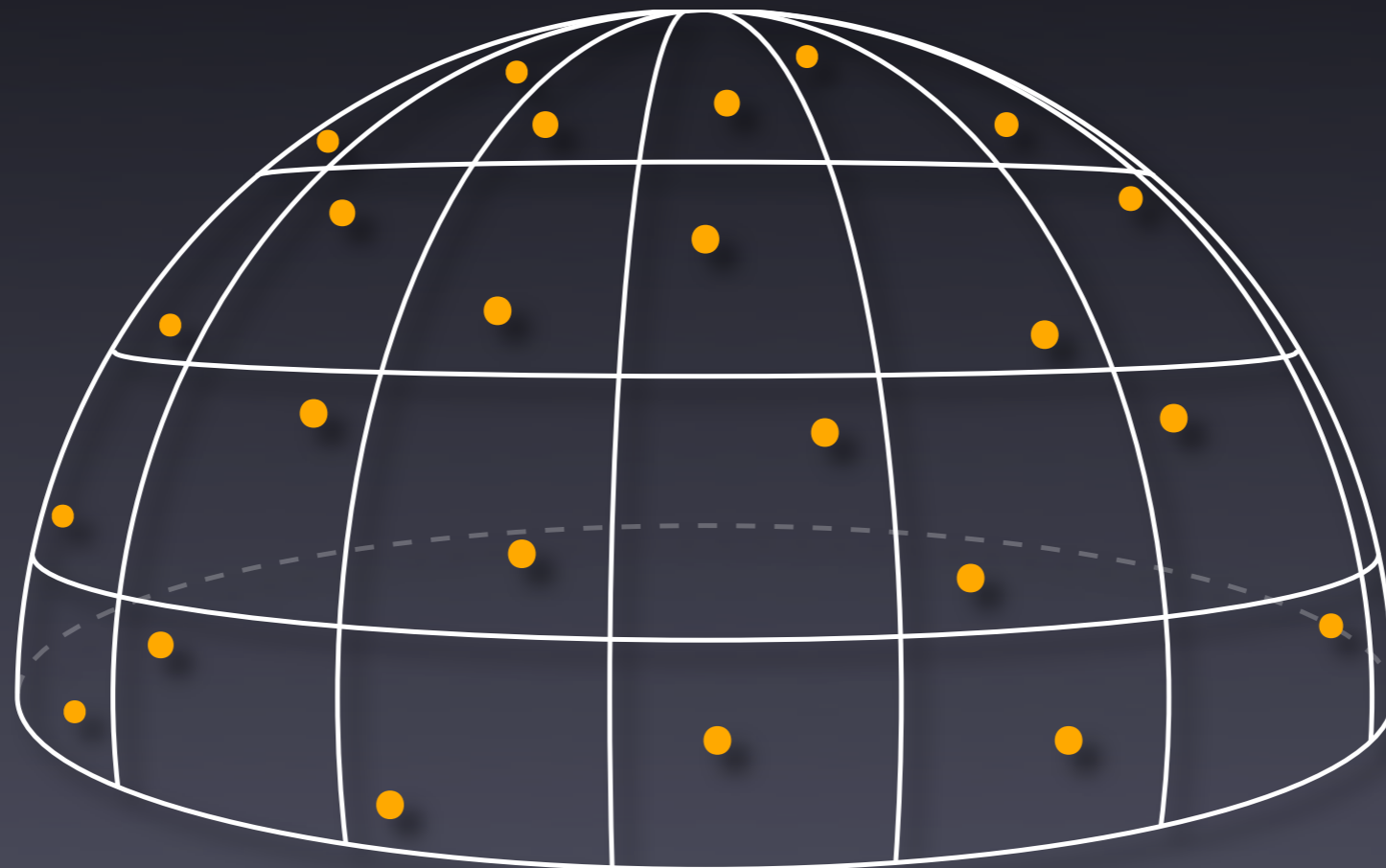
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Ward et al. '88



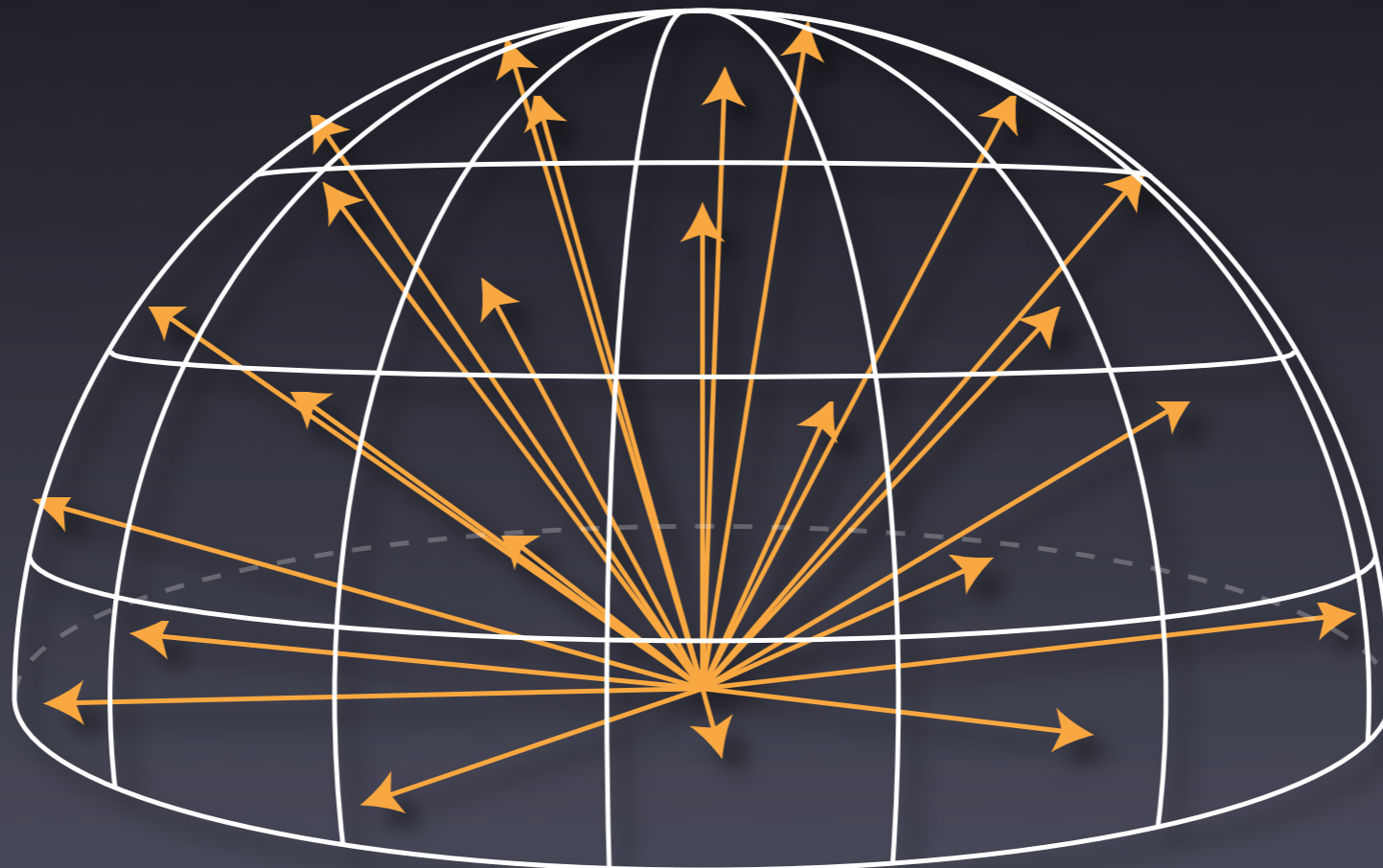
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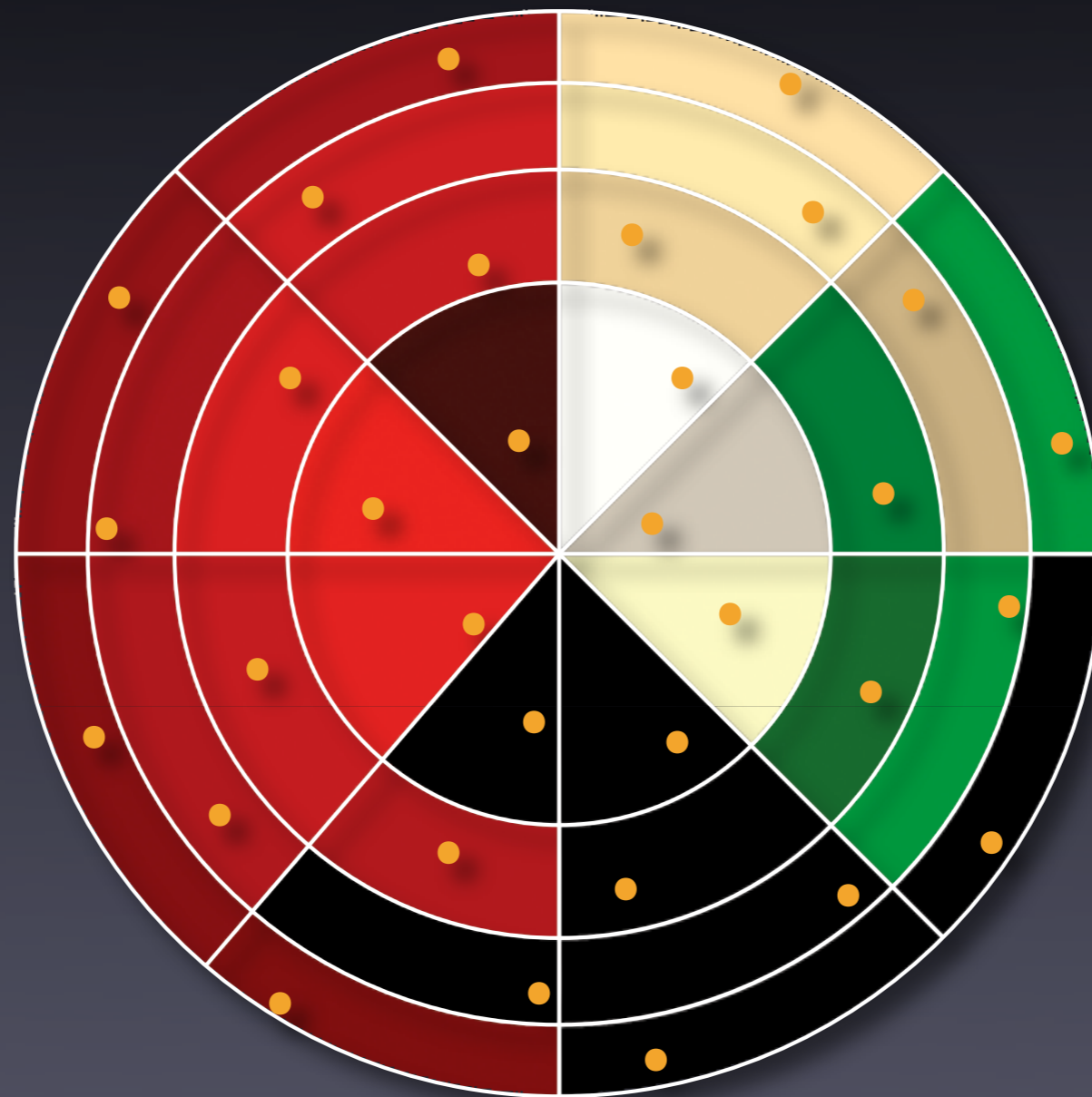
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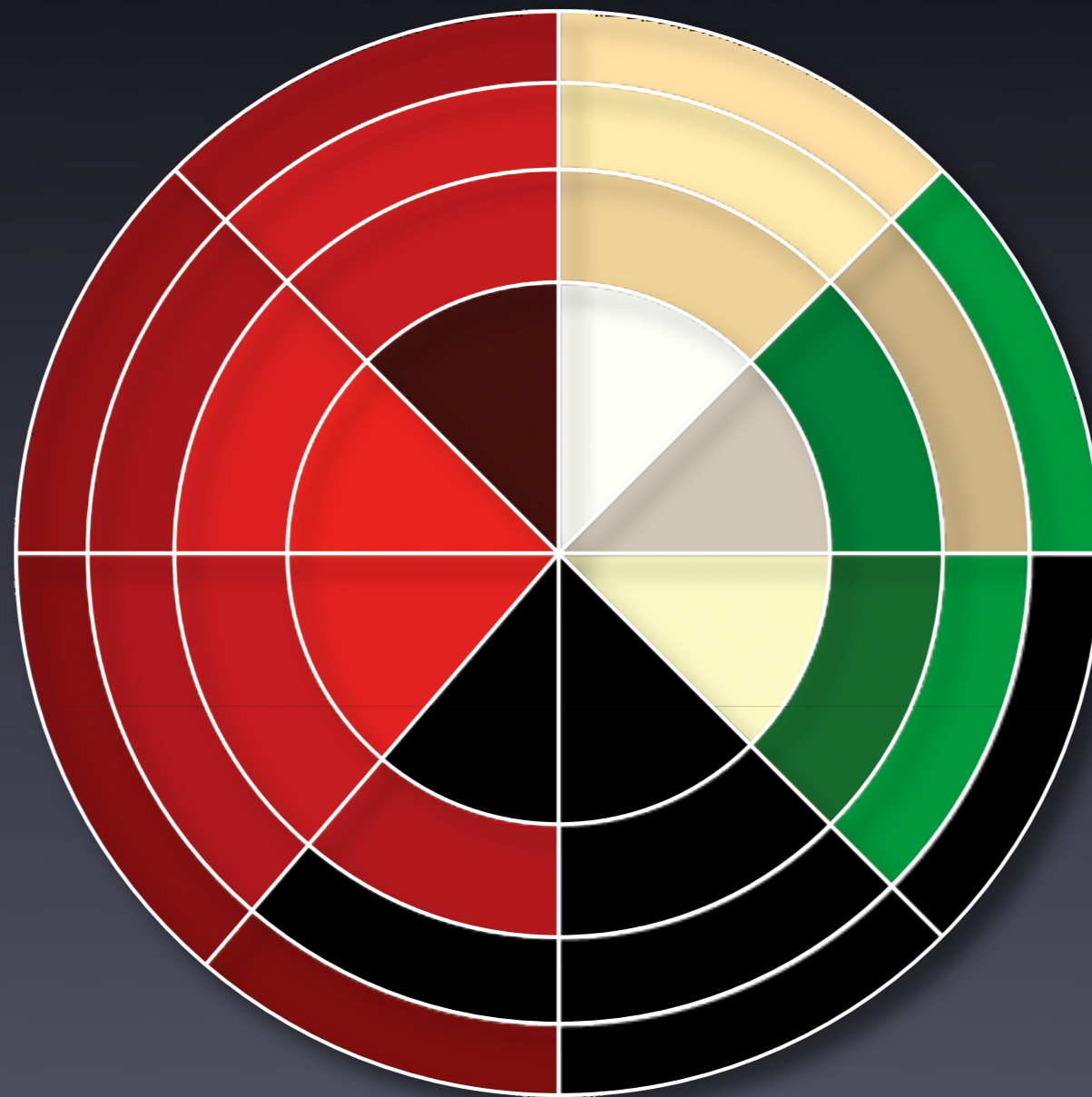
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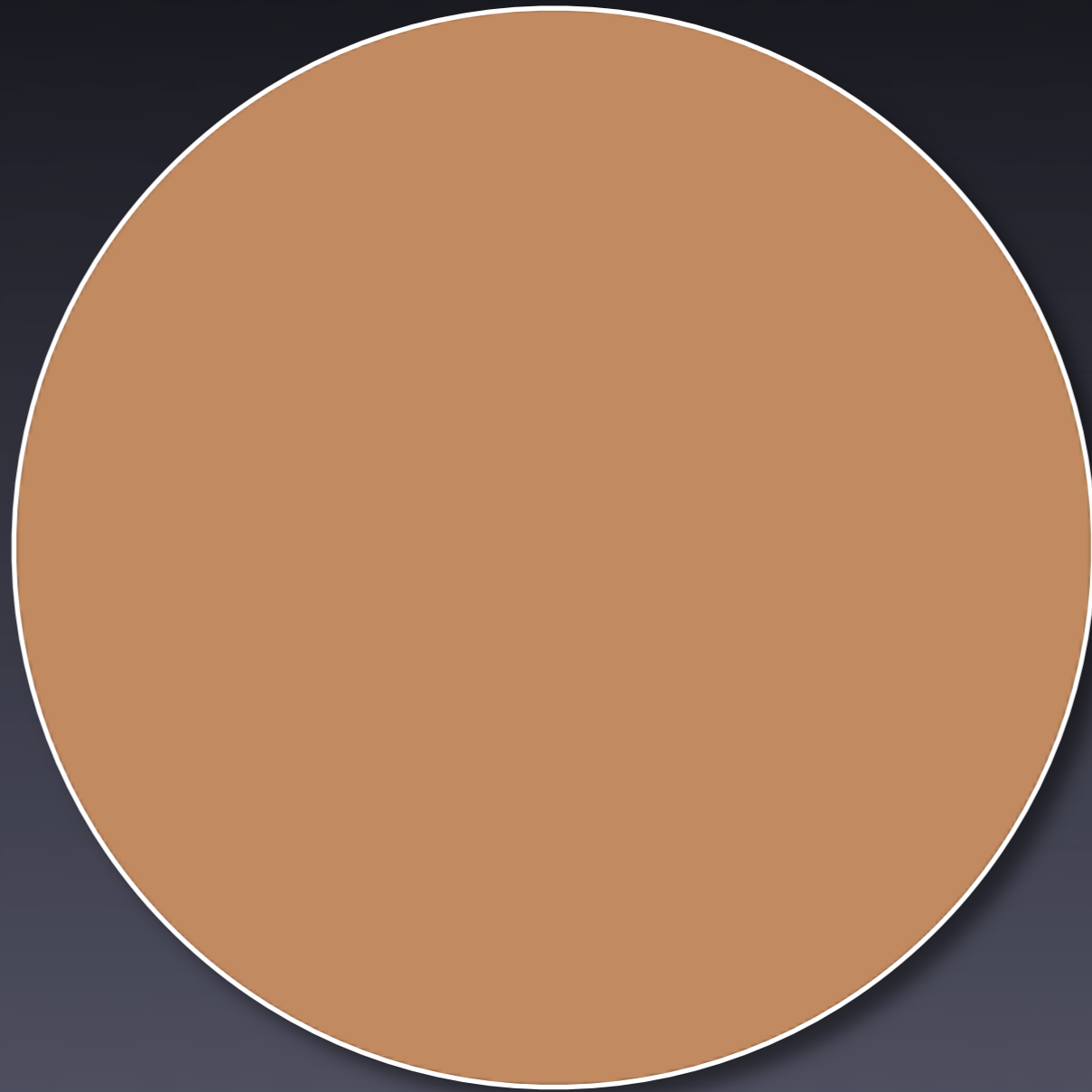
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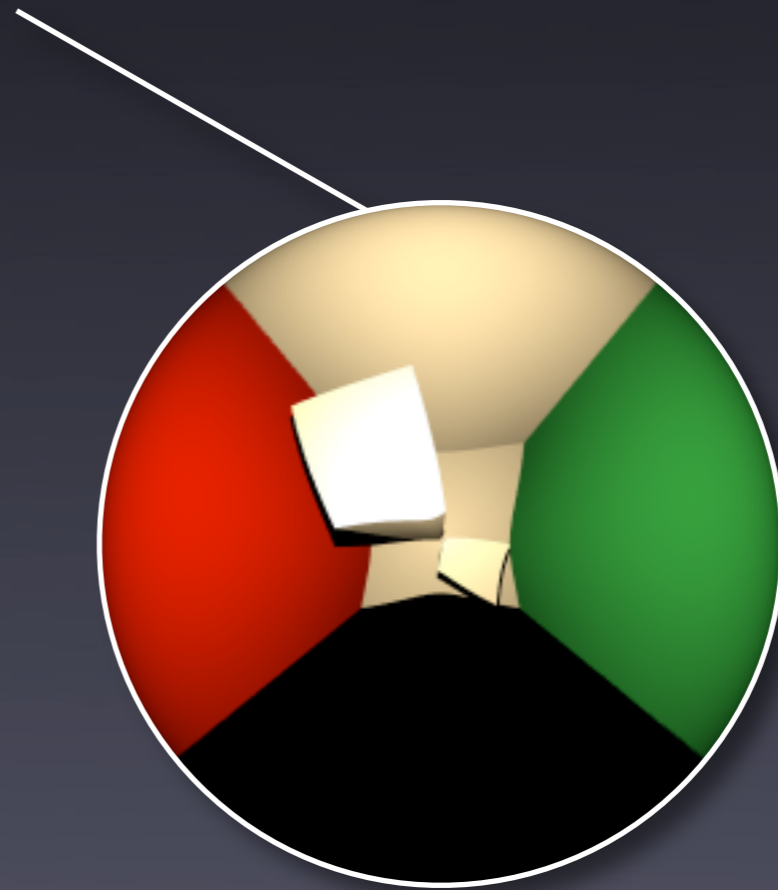
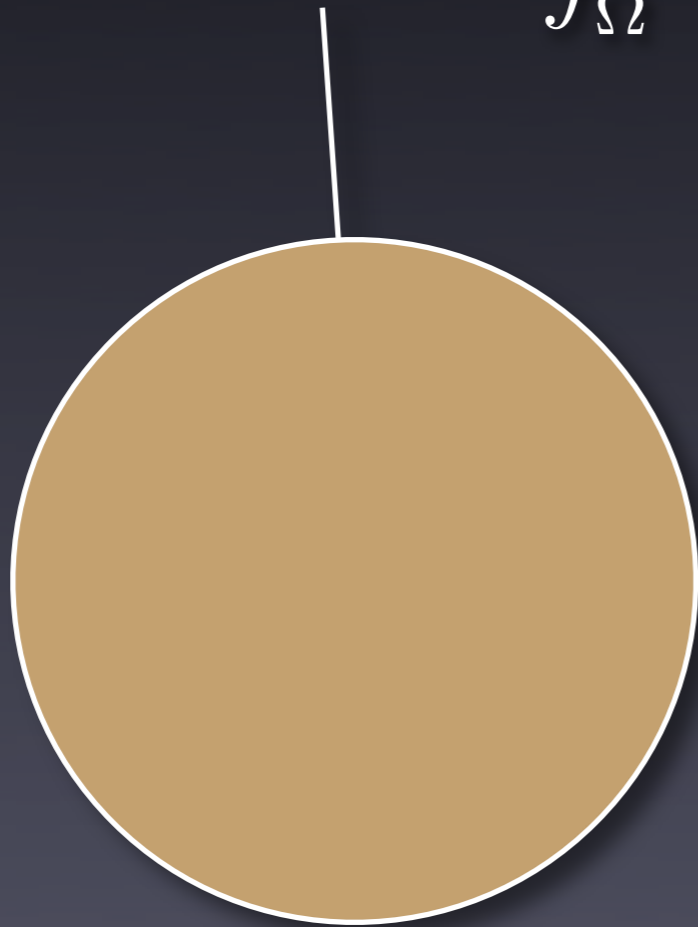
Ward et al. '88



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Ward et al. '88

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Ward et al. '88

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# Computing Irradiance

Ward et al. '88

$$E(\mathbf{x}) = \int_{\Omega} L_i(\mathbf{x}, \vec{\omega}_i) (\vec{\omega}_i \cdot \vec{n}) d\vec{\omega}_i$$

$$E(\mathbf{x}) \approx \frac{\pi}{N_1 N_2} \sum_{j=1}^{N_1} \sum_{i=1}^{N_2} L_i(\mathbf{x}, \theta_j, \phi_i),$$

where:

$$\theta_j = \sin^{-1} \left( \sqrt{\frac{j - \xi_1}{N_1}} \right), \quad \phi_i = 2\pi \frac{i - \xi_2}{N_2}$$

# Interpolating Irradiance

Ward et al. '88

- We wish to minimize the number of times we compute this costly irradiance integral.
- How far away can we reuse a cache point?

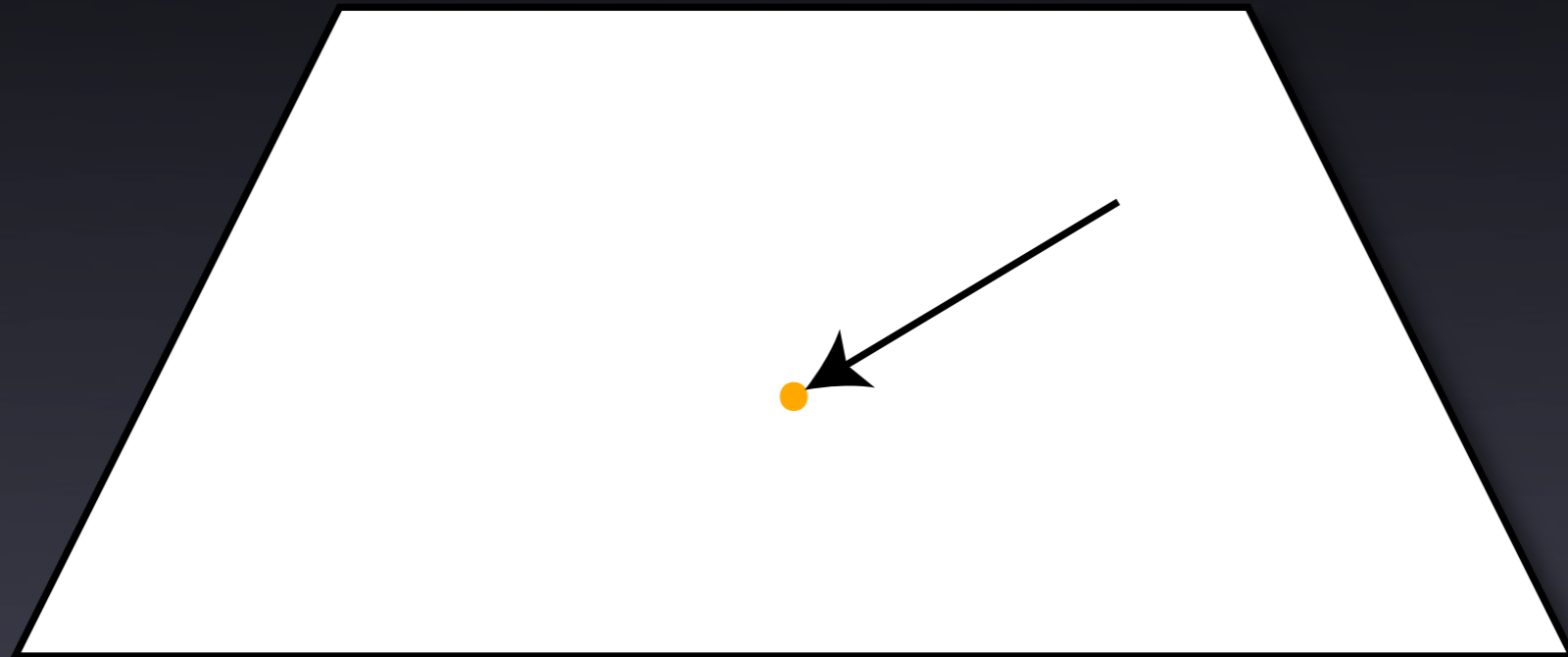
# Interpolating Irradiance

Ward et al. '88

- Ward et al. derived an approximate formula for this error function.
- Error function is decomposed w.r.t. translation and orientation.
- Translation: function of distance to cache point & average distance to surface visible at cache point.
- Orientation: function of rotational offset

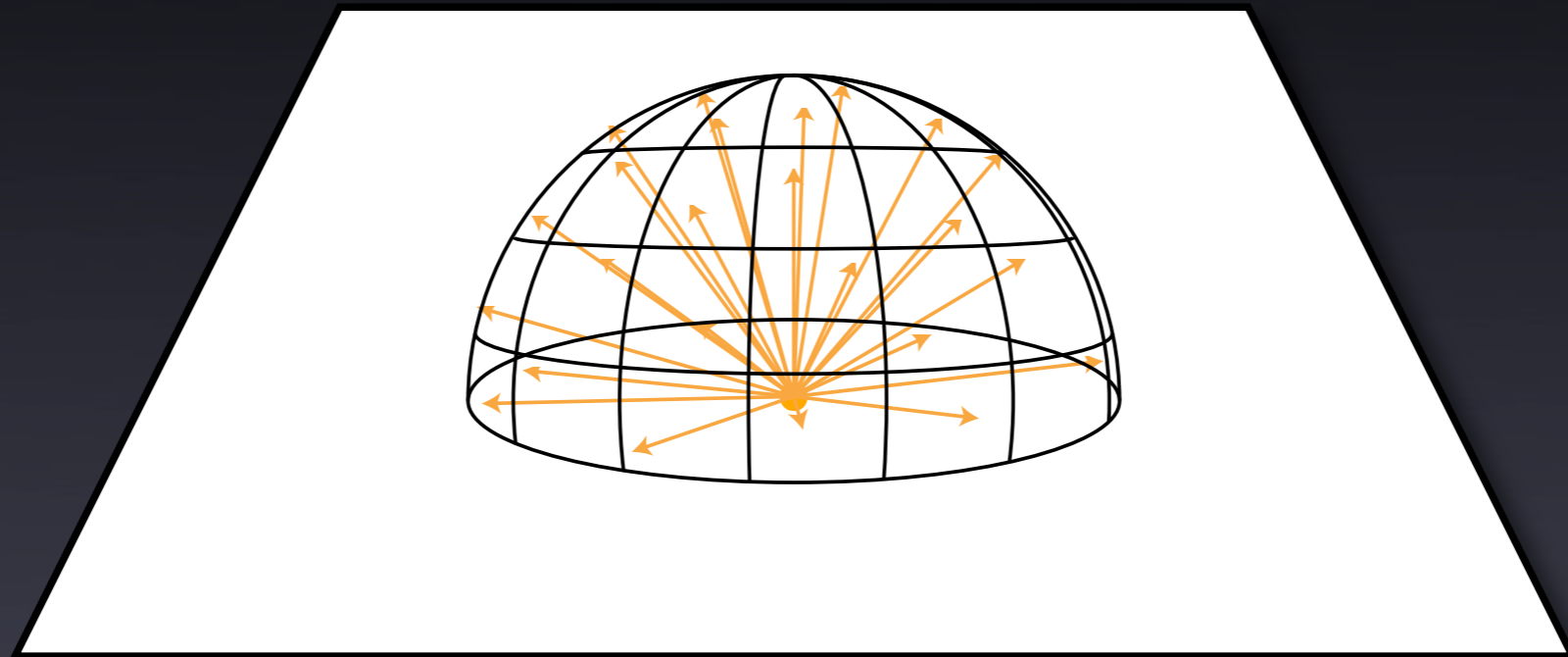
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Ward et al. '88



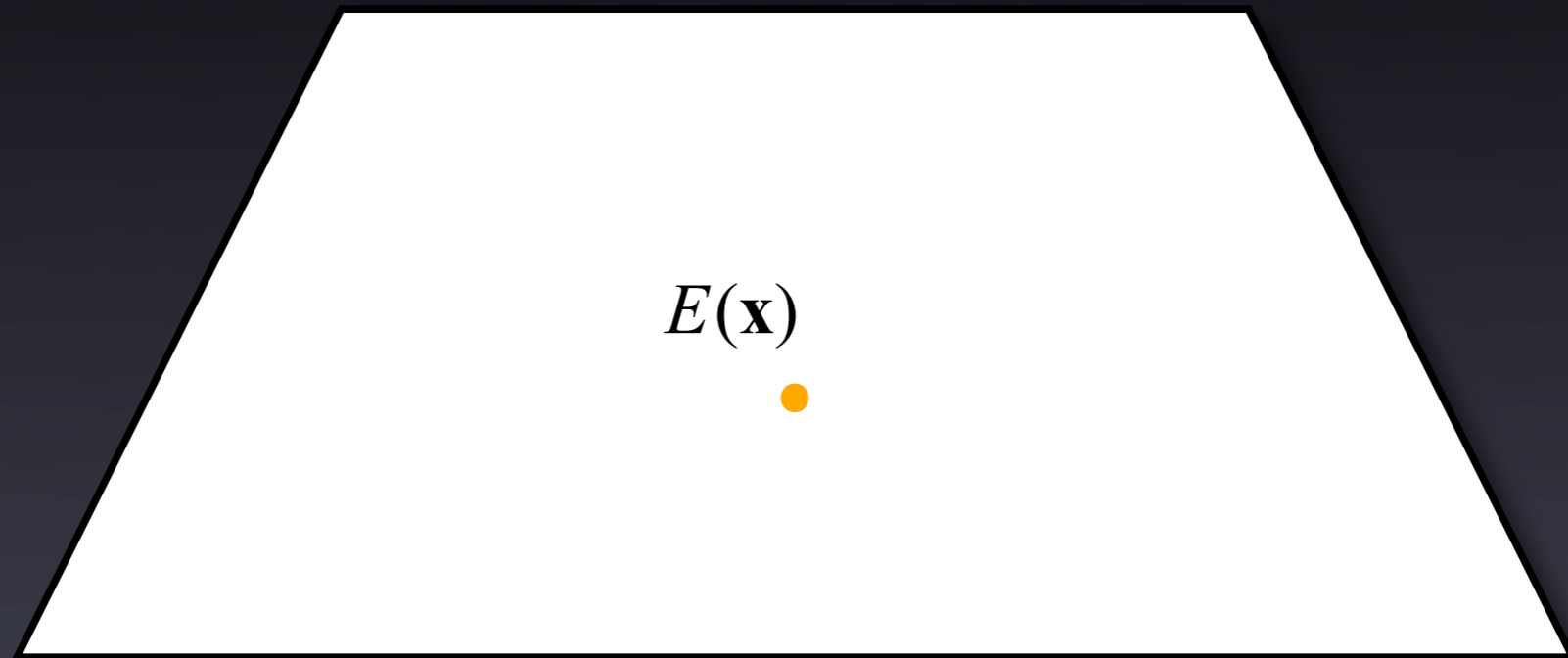
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Ward et al. '88



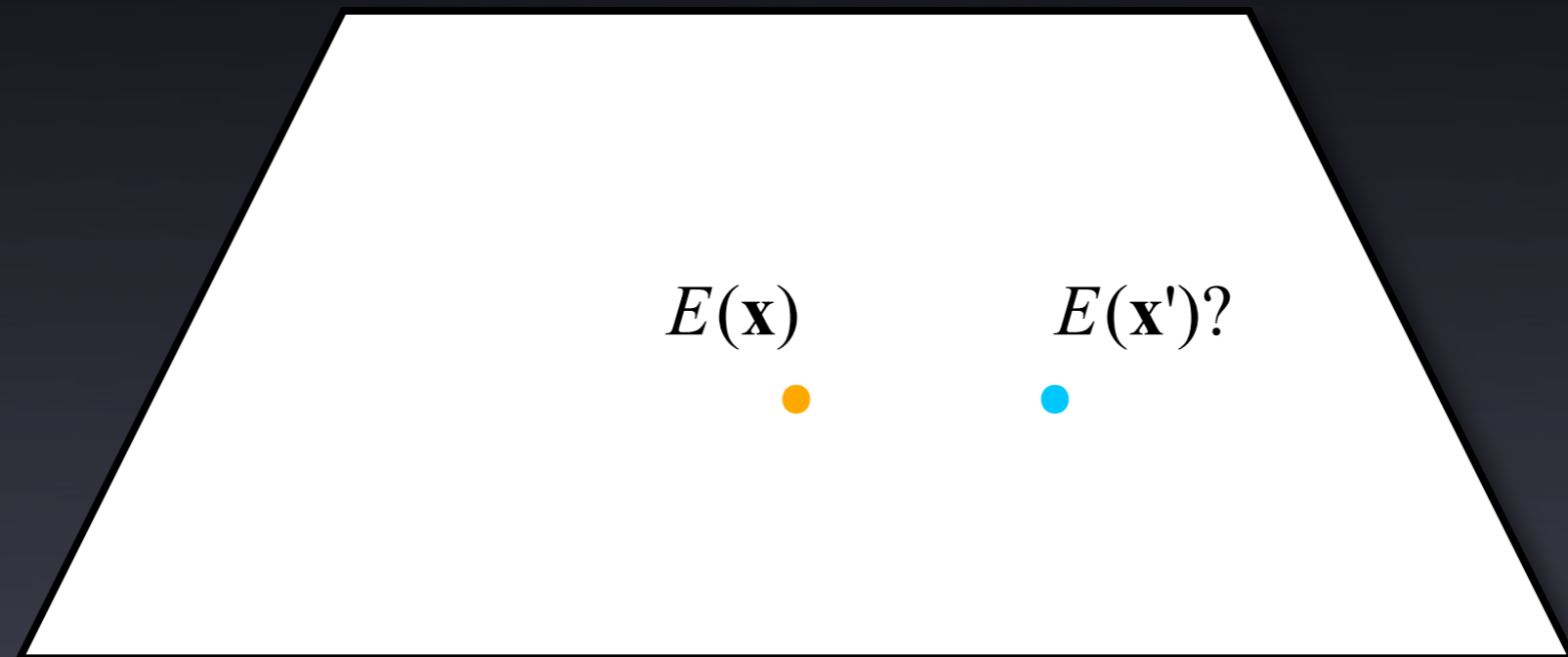
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Ward et al. '88



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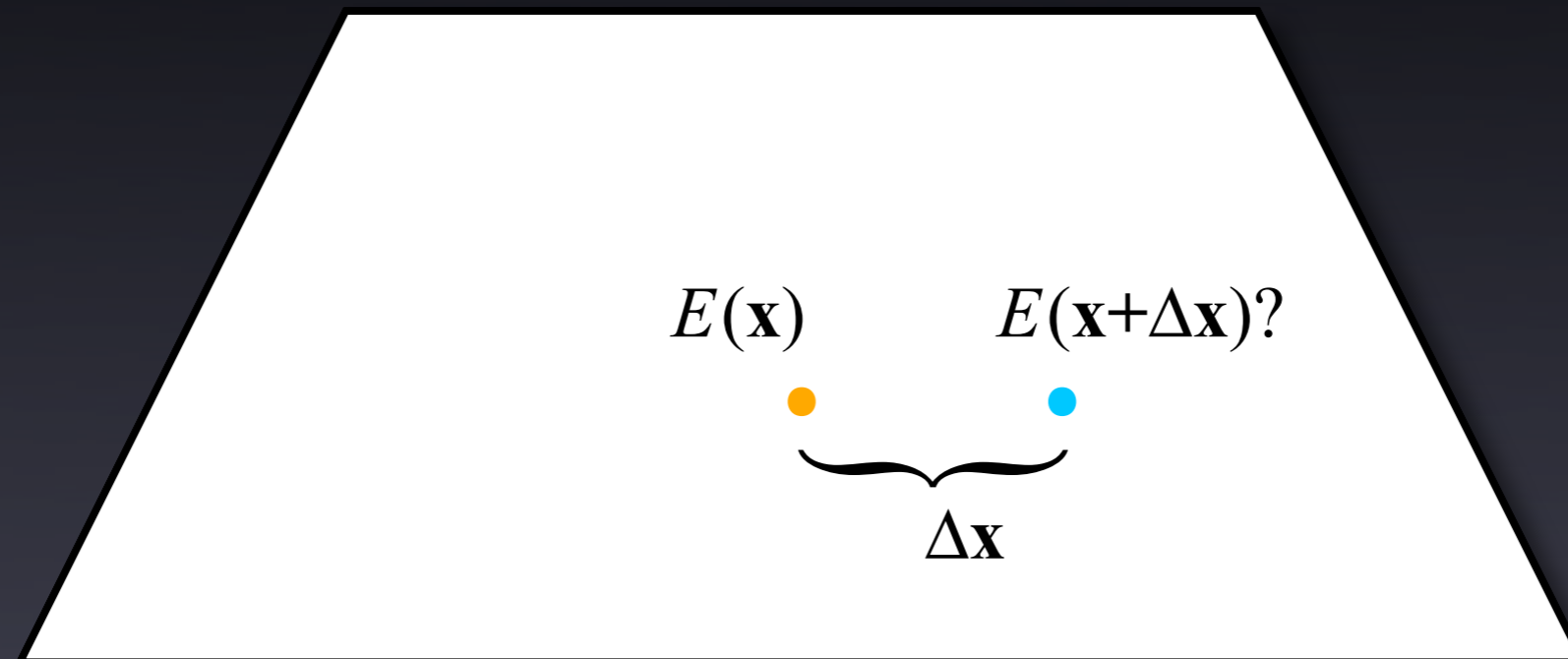
Ward et al. '88



What happens if we re-use  $E(\mathbf{x})$  at  $E(\mathbf{x}')$ ?

# Interpolating Irradiance

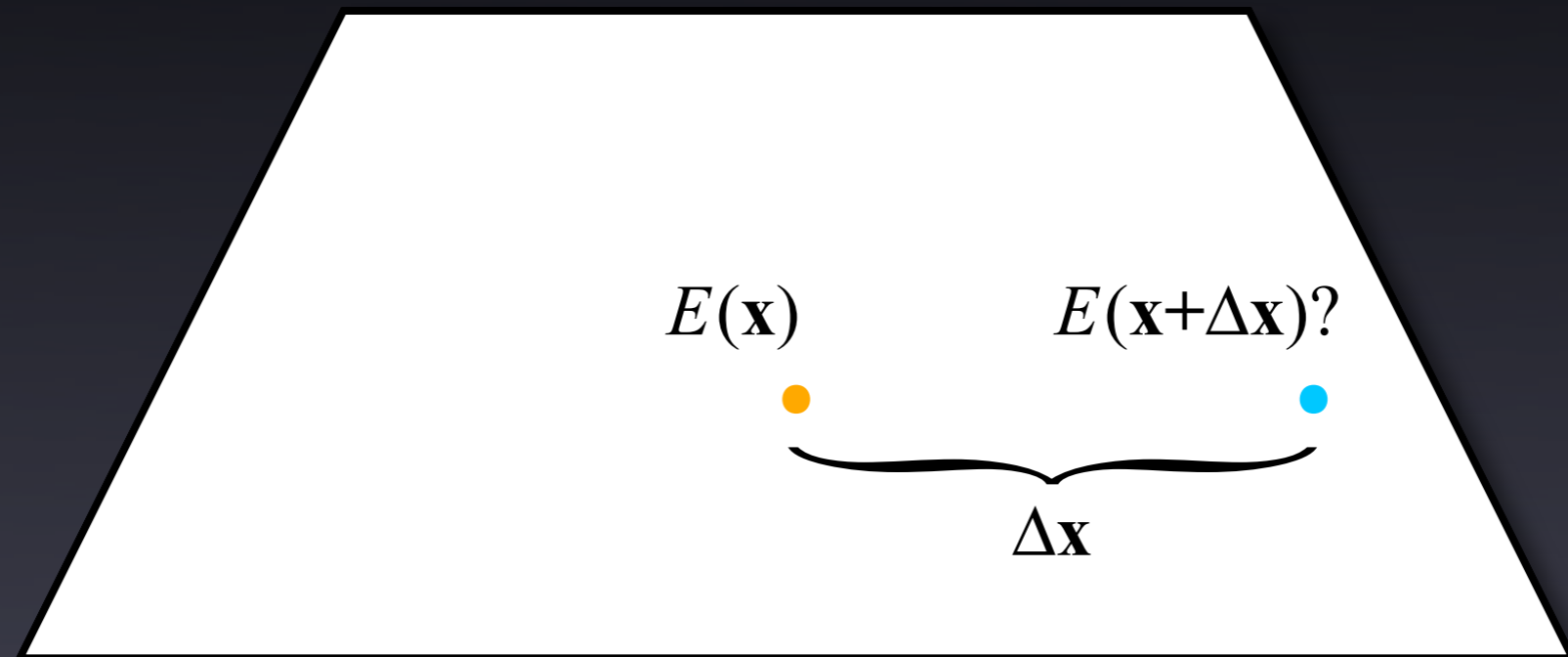
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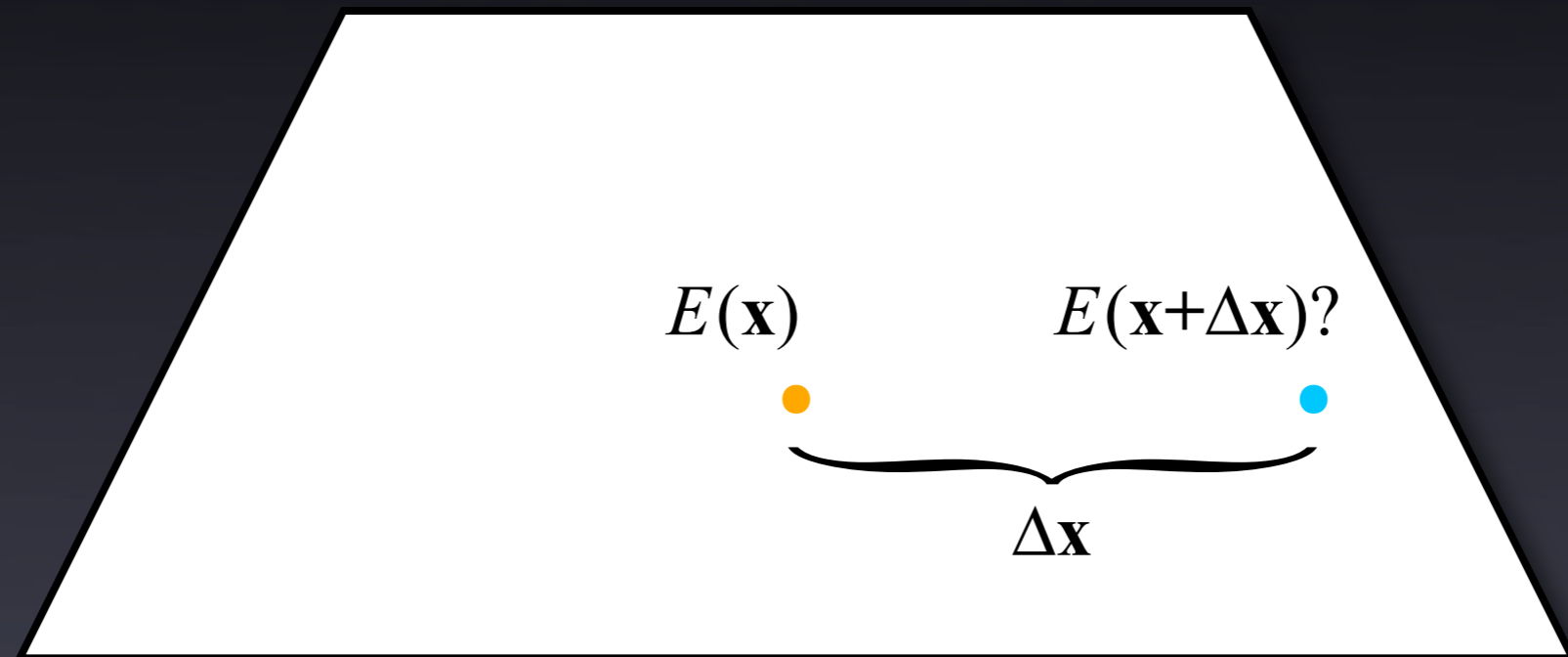
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# Interpolating Irradiance

Ward et al. '88

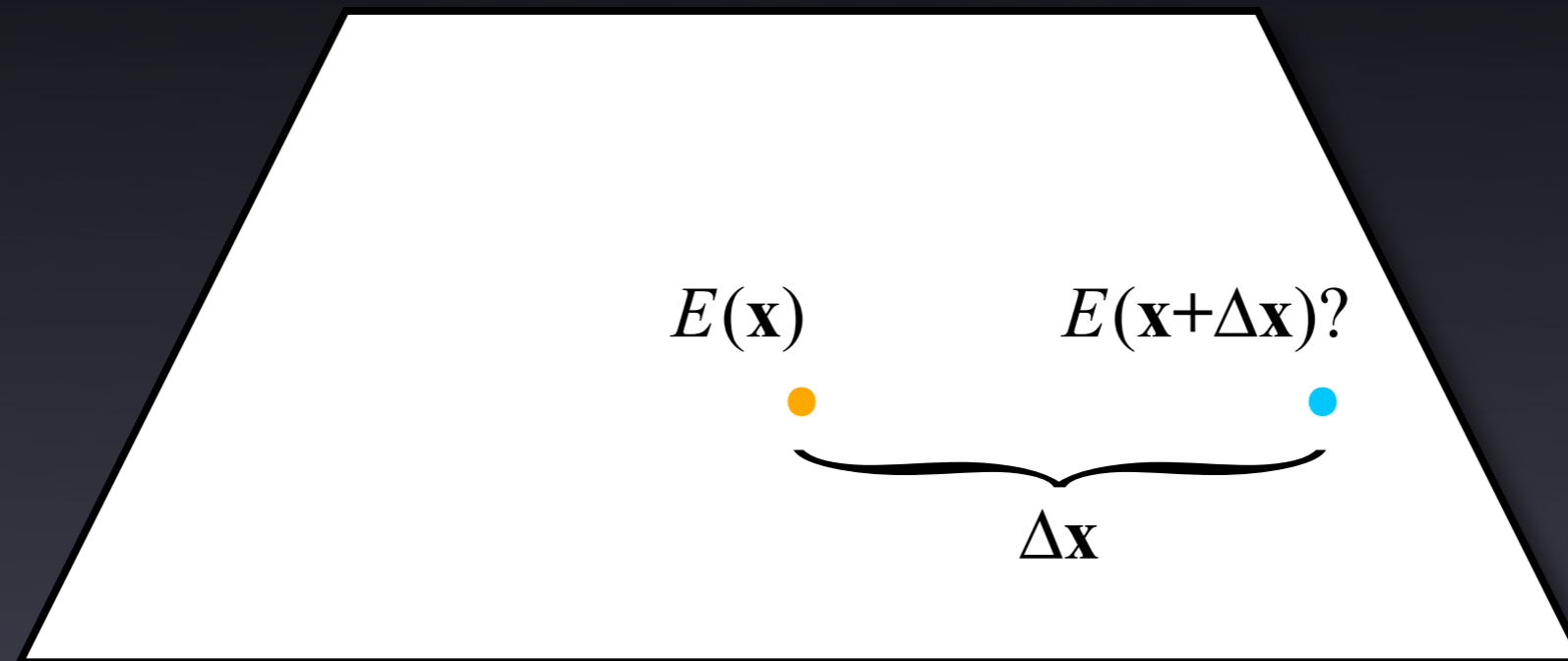


What happens if we re-use  $E(\mathbf{x})$  at  $E(\mathbf{x}')$ ?

$$E(x') \approx E(x) + \left( \frac{\partial E}{\partial x}(\Delta x) + \frac{\partial E}{\partial \varphi}(\Delta \varphi) \right)$$

# Interpolating Irradiance

Ward et al. '88



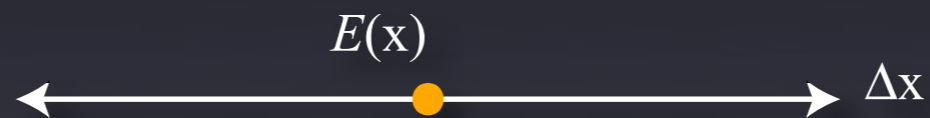
What happens if we re-use  $E(\mathbf{x})$  at  $E(\mathbf{x}')$ ?

error

$$E(x') \approx E(x) + \left( \frac{\partial E}{\partial x}(\Delta x) + \frac{\partial E}{\partial \varphi}(\Delta \varphi) \right)$$

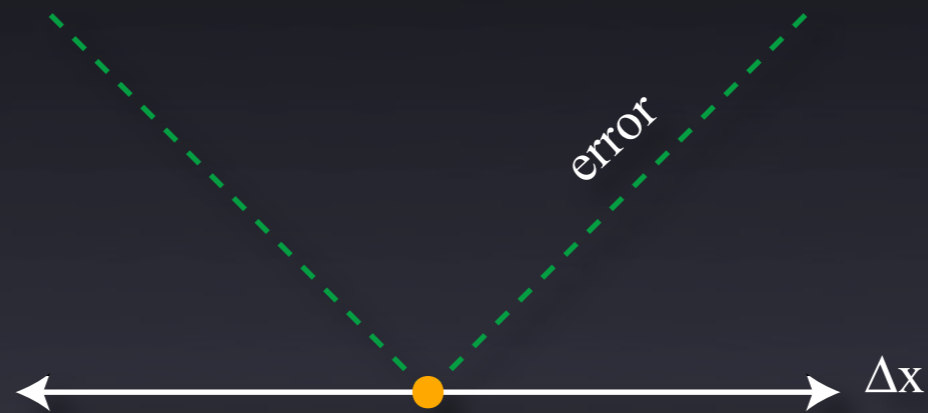
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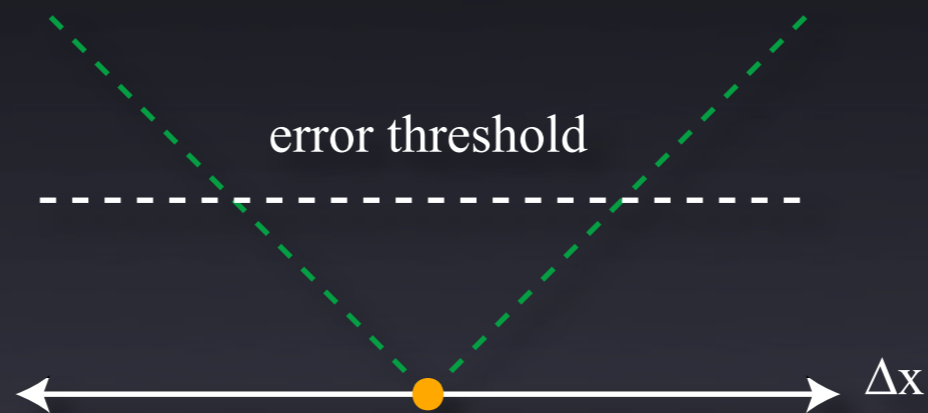
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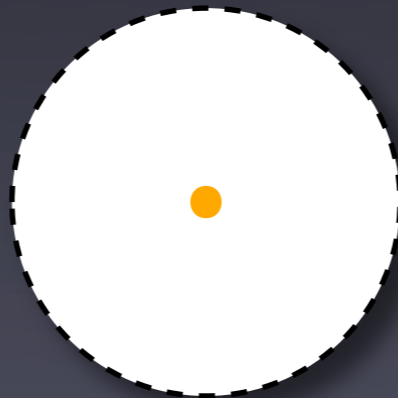
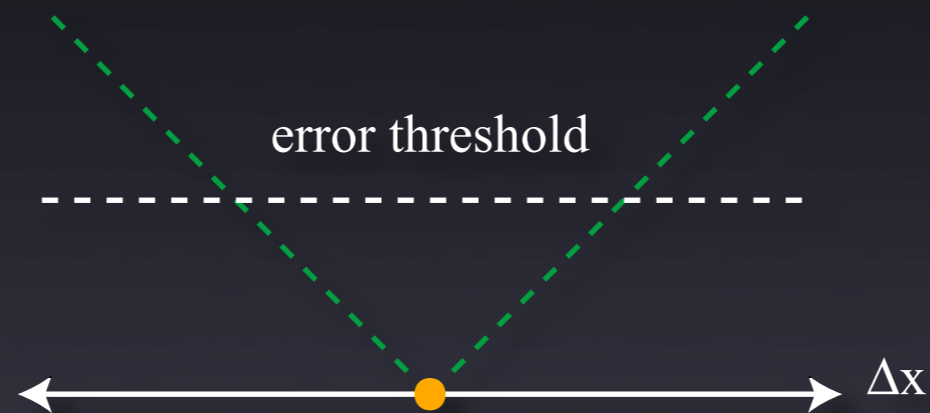
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Ward et al. '88



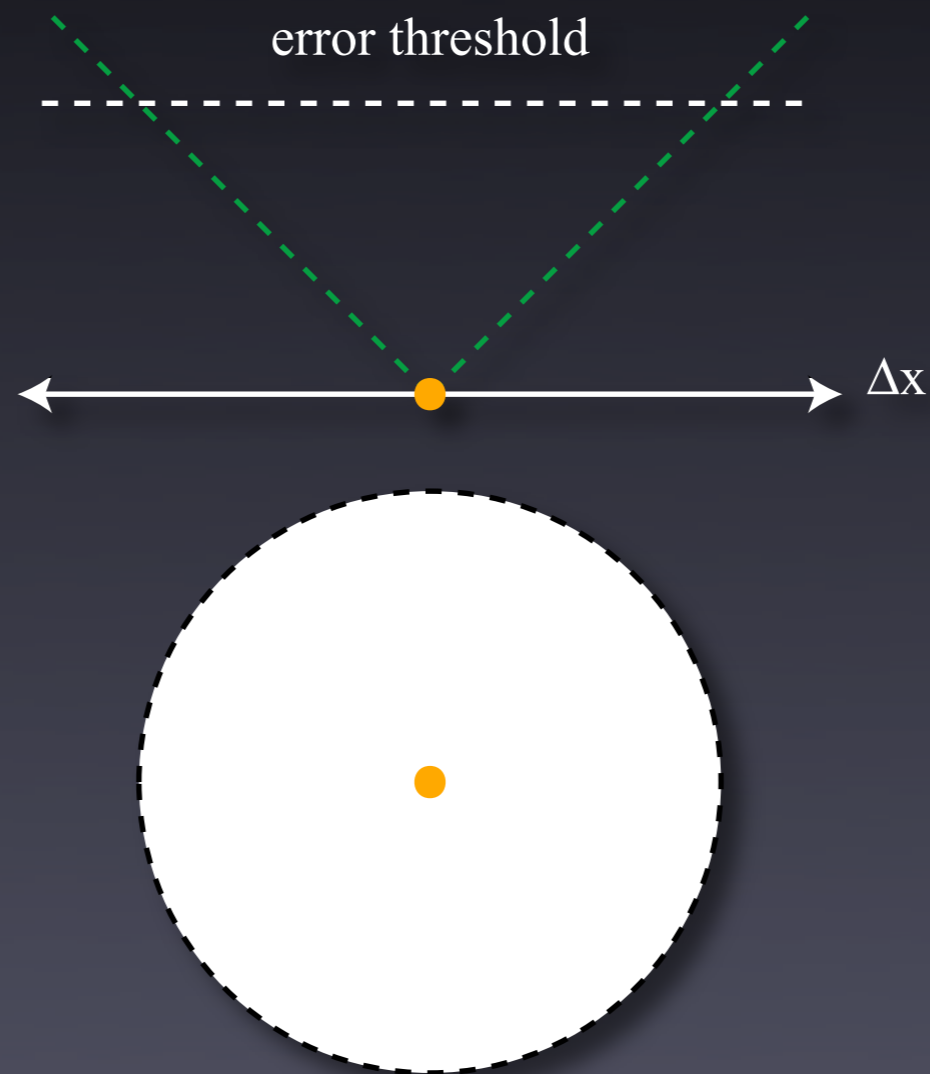
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Ward et al. '88



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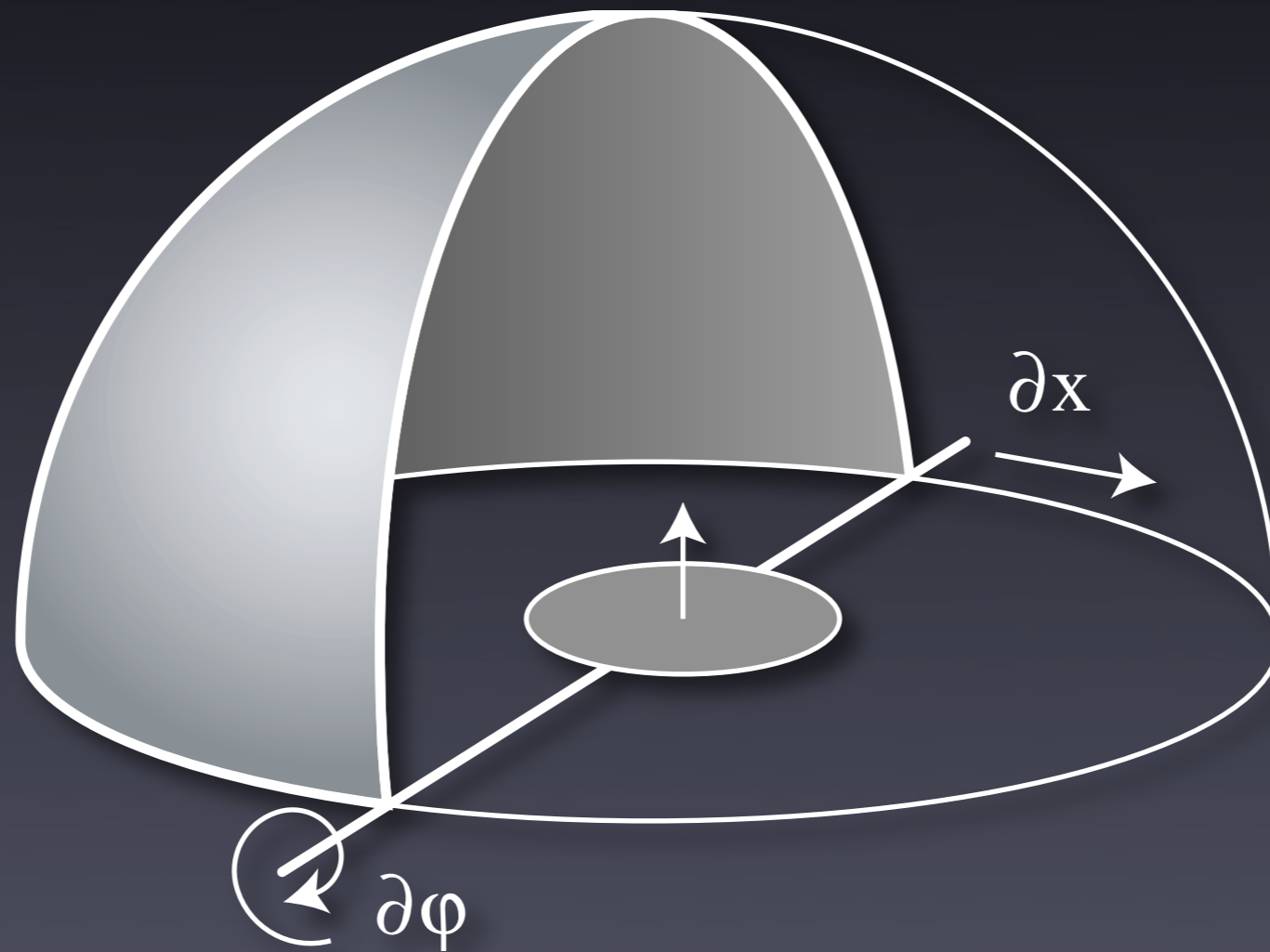
Ward et al. '88

- Approximate using Taylor expansion:

$$\varepsilon \lesssim \left| \frac{\partial E}{\partial x} (x - x_0) + \frac{\partial E}{\partial \varphi} (\varphi - \varphi_0) \right|$$

# The “Split-Sphere”

Ward et al. '88



# Interpolating Irradiance

Ward et al. '88

- In the “Split-Sphere” environment the error becomes:

$$\varepsilon \lesssim \left| \frac{\partial E}{\partial x}(x - x_0) + \frac{\partial E}{\partial \varphi}(\varphi - \varphi_0) \right|$$

# Interpolating Irradiance

Ward et al. '88

- In the “Split-Sphere” environment the error becomes:

$$\varepsilon \lesssim E_0 \left( \frac{4}{\pi} \frac{|x - x_0|}{R} + |\varphi - \varphi_0| \right)$$

# Interpolating Irradiance

Ward et al. '88

- In the “Split-Sphere” environment the error becomes:

$$\varepsilon \lesssim E_0 \left( \frac{4}{\pi} \frac{|x - x_0|}{\boxed{R}} + |\varphi - \varphi_0| \right)$$

“average” distance to visible surfaces in hemisphere

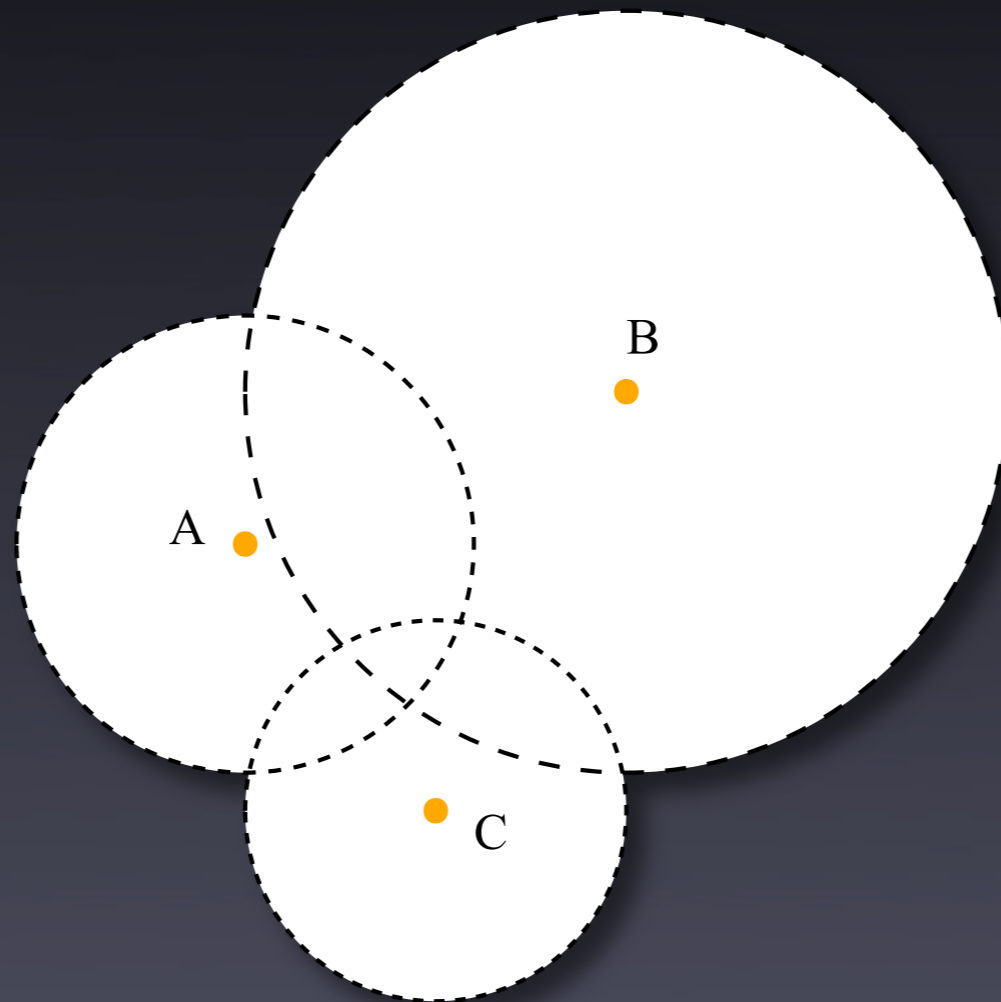
# Interpolating Irradiance

Ward et al. '88

- At each shading location, perform a weighted average of all cached values which have an error below some threshold.
- Reciprocal of the error is used as the weight.

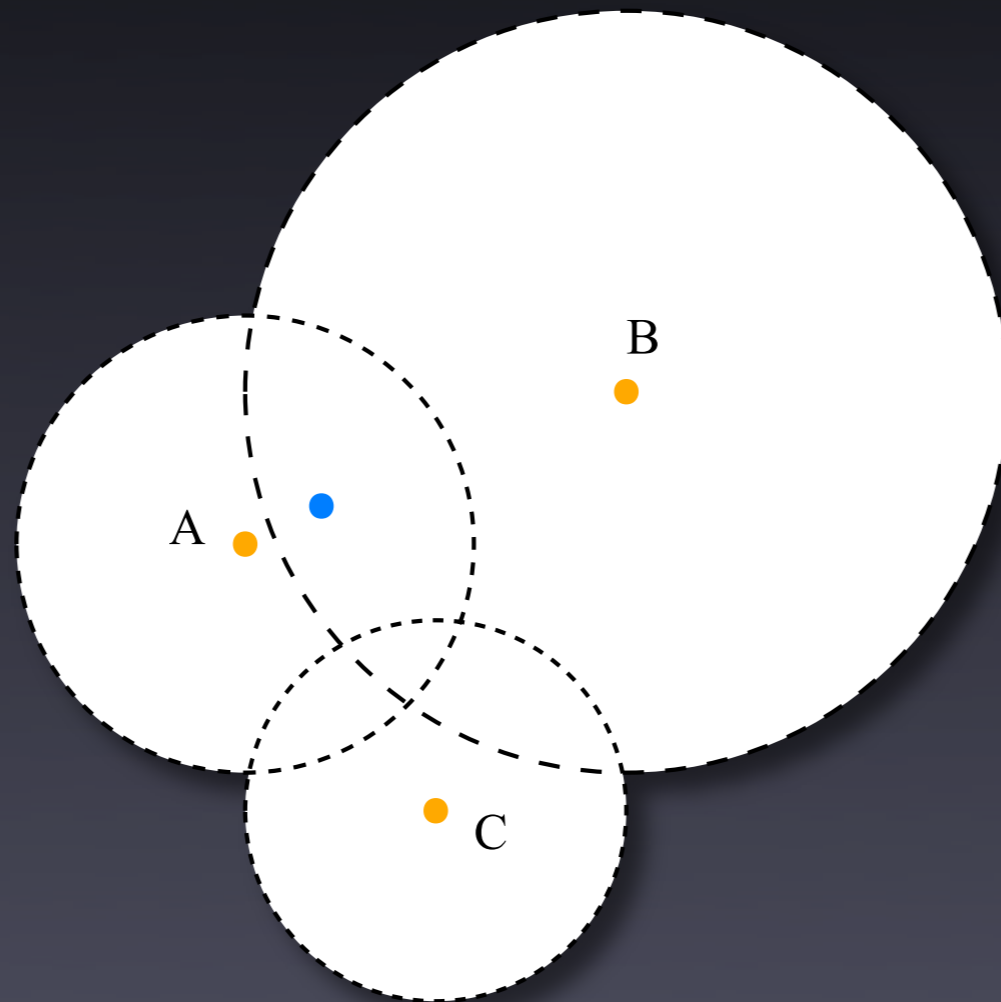
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Ward et al. '88



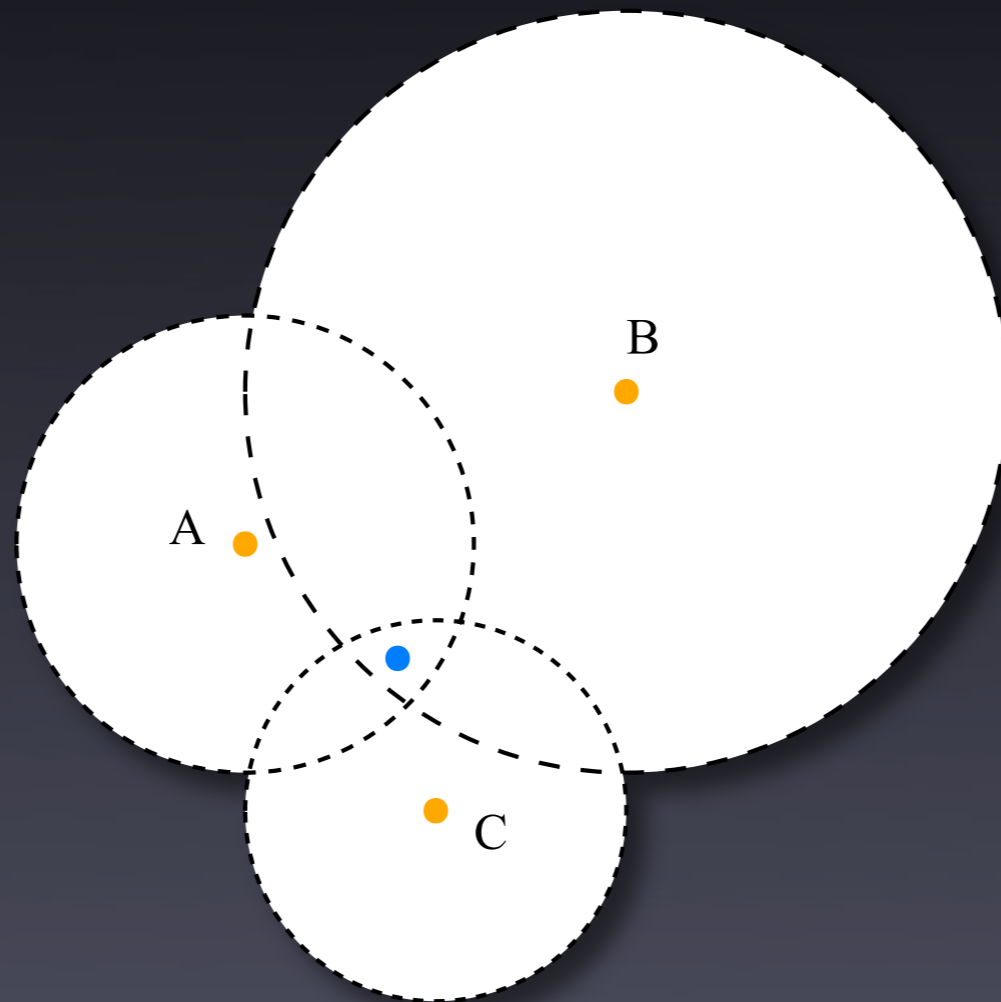
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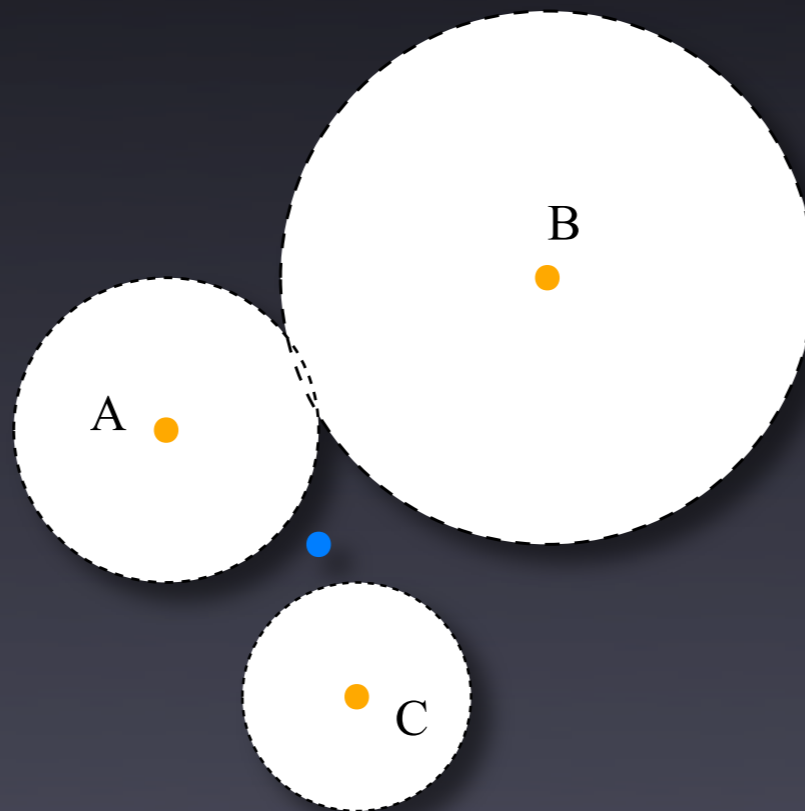
# Interpolating Irradiance

Ward et al. '88



# Interpolating Irradiance

Ward et al. '88



# Interpolating Irradiance

Ward et al. '88

$$E(\mathbf{x}, \vec{n}) \approx \frac{\sum_{i \in S} w_i(\mathbf{x}, \vec{n}) E_i}{\sum_{i \in S} w_i(\mathbf{x}, \vec{n})}$$

# Interpolating Irradiance

Ward et al. '88

$$E(\mathbf{x}, \vec{n}) \approx \frac{\sum_{i \in S} w_i(\mathbf{x}, \vec{n}) E_i}{\sum_{i \in S} w_i(\mathbf{x}, \vec{n})}$$

where:

$$w_i(\mathbf{x}, \vec{n}) = \frac{1}{\frac{\|\mathbf{x} - \mathbf{x}_i\|}{R_i} + \sqrt{1 - \vec{n} \cdot \vec{n}_i}}$$

$$S = \left\{ i : w_i(\mathbf{x}, \vec{n}) > \frac{1}{a} \right\}$$

# Irradiance Caching

Ward et al. '88

- Pros:
  - Independent of resolution.
  - Concentrates computation in visible regions.
  - Hundreds of times faster than naive path tracing.

# Irradiance Caching

Ward et al. '88

- Cons:
  - Interpolation/extrapolation can introduce visible artifacts.
  - Limited to Lambertian (matte) surfaces.
  - Still slow.

# Improvements/Extensions

- Ward and Heckbert '92 - better interpolation
- Křivánek et al. '05a, '05b - glossy surfaces
- Tabellion and Lamorlette '04 - speed
- Tawara et al. '04 - animation
- Yee '00 - speed/perception
- Kato '02 - parallel/distributed computation
- Arikan et al. '05 - speed

# Improvements/Extensions

- Ward and Heckbert '92
- Křivánek et al. '05a, '05b
- Tabellion and Lamorlette '04
- Tawara et al. '04

# Improvements/Extensions

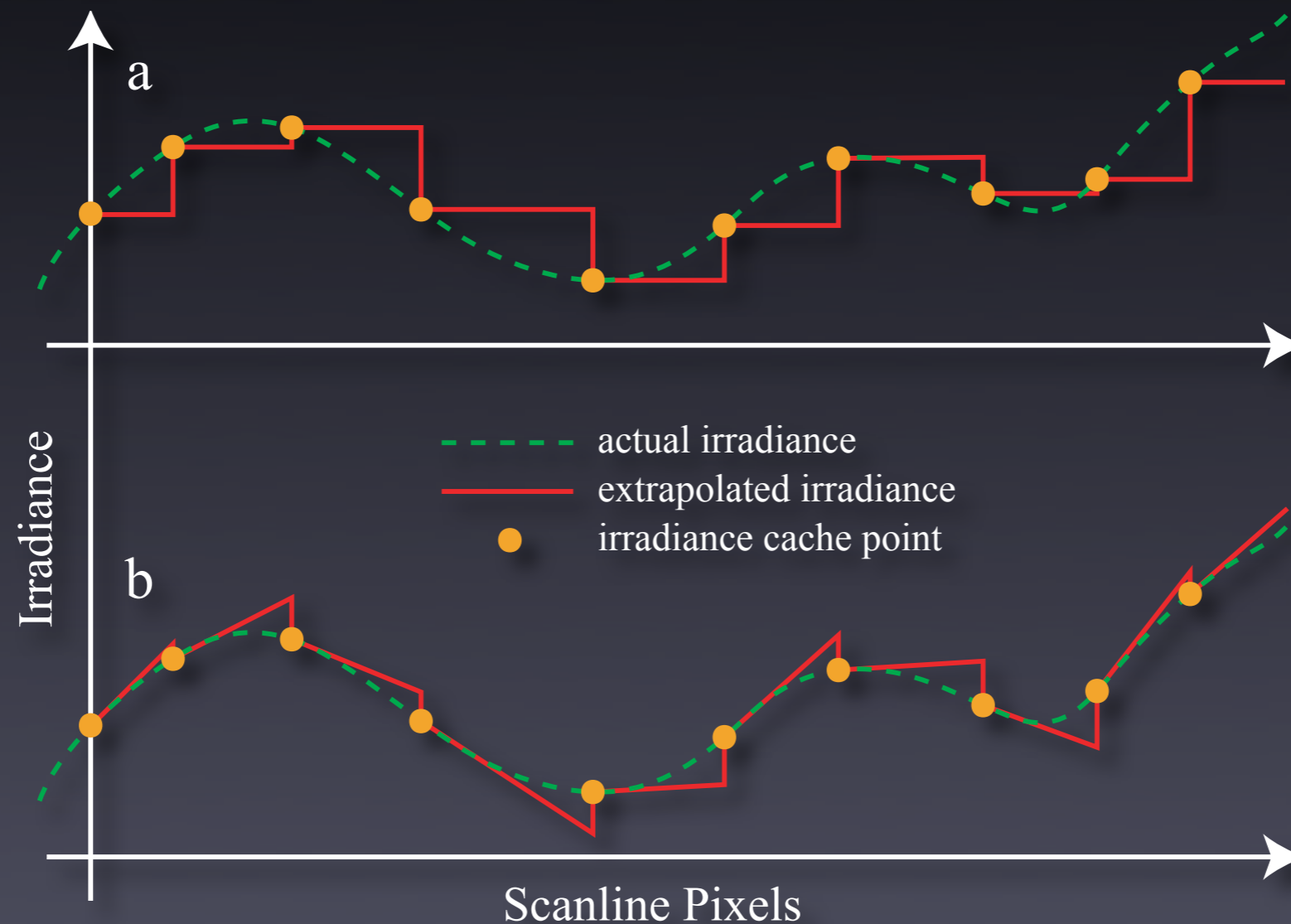
- Ward and Heckbert '92
- Křivánek et al. '05a, '05b
- Tabellion and Lamorlette '04
- Tawara et al. '04

# Irradiance Gradients

Ward and Heckbert '92

- Improve quality by performing higher-order interpolation/extrapolation.
- Estimate a derivative to the irradiance.
- Apply this derivative to the weighted average.

# Extrapolating Irradiance



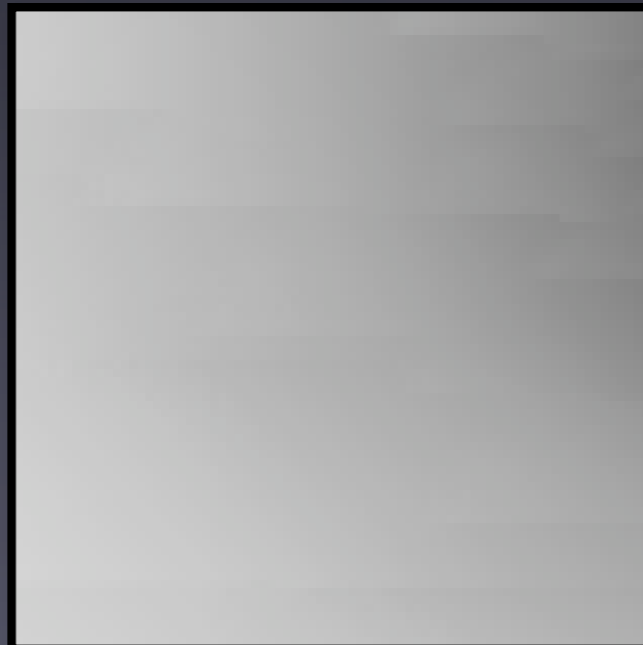
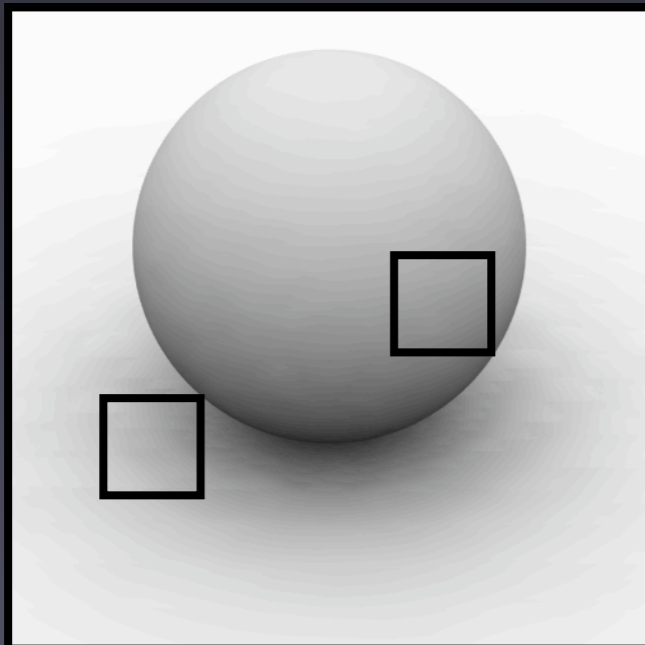
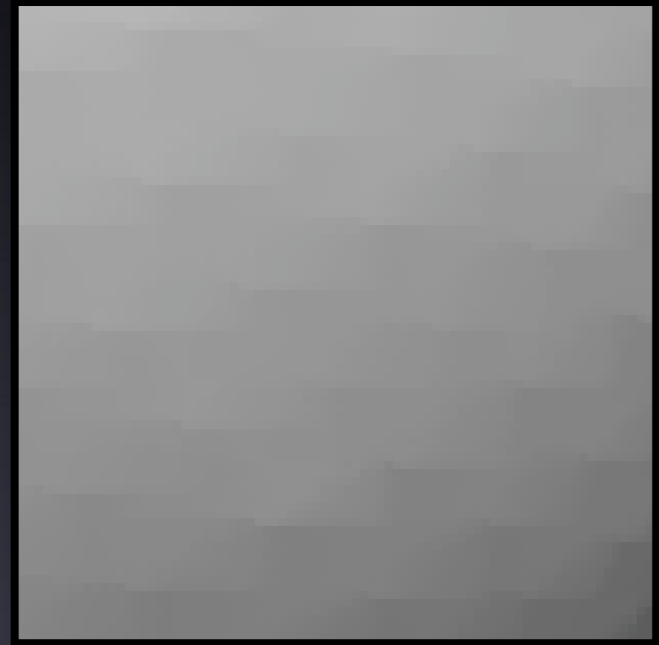
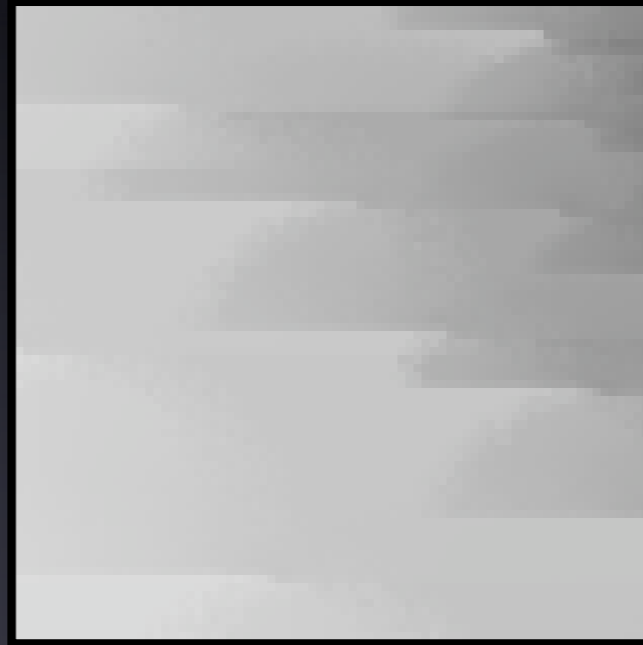
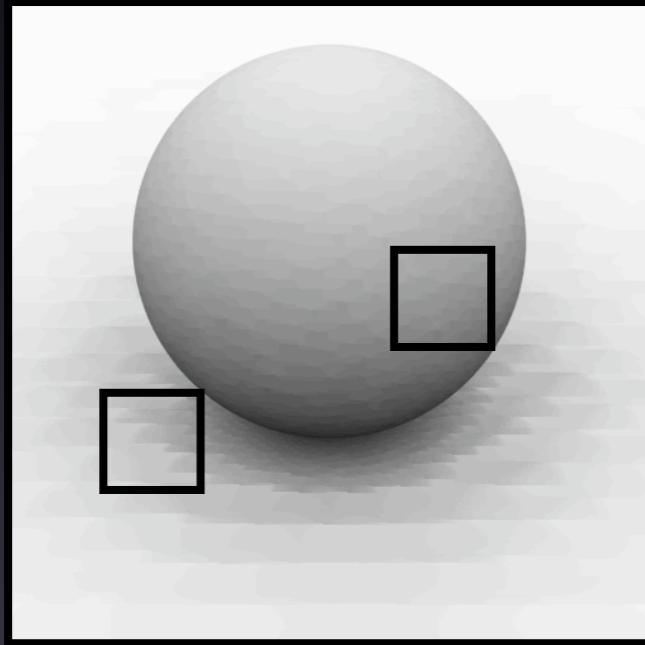
# Irradiance Gradients

Ward and Heckbert '92

- Decomposed into:
  - Rotational Gradient - captures change in irradiance w.r.t. change in surface orientation
  - Translational Gradient - captures change in irradiance w.r.t. change in position

# Irradiance Gradients

Ward and Heckbert '92



# Applying Gradients

Ward and Heckbert '92

$$E(\mathbf{x}, \vec{n}) \approx \frac{\sum_{i \in S} w_i(\mathbf{x}, \vec{n}) E_i}{\sum_{i \in S} w_i(\mathbf{x}, \vec{n})}$$

# Applying Gradients

Ward and Heckbert '92

$$E(\mathbf{x}, \vec{n}) \approx \frac{\sum_{i \in S} w_i(\mathbf{x}, \vec{n}) \left( E_i + (\vec{n}_i \times \vec{n}) \cdot \vec{\nabla}_r E_i + (\mathbf{x} - \mathbf{x}_i) \cdot \vec{\nabla}_t E_i \right)}{\sum_{i \in S} w_i(\mathbf{x}, \vec{n})}$$

# Applying Gradients

Ward and Heckbert '92

estimated change due to rotation and translation

$$E(\mathbf{x}, \vec{n}) \approx \frac{\sum_{i \in S} w_i(\mathbf{x}, \vec{n}) \left( E_i + \boxed{(\vec{n}_i \times \vec{n}) \cdot \vec{\nabla}_r E_i} + \boxed{(\mathbf{x} - \mathbf{x}_i) \cdot \vec{\nabla}_t E_i} \right)}{\sum_{i \in S} w_i(\mathbf{x}, \vec{n})}$$

# Improvements/Extensions

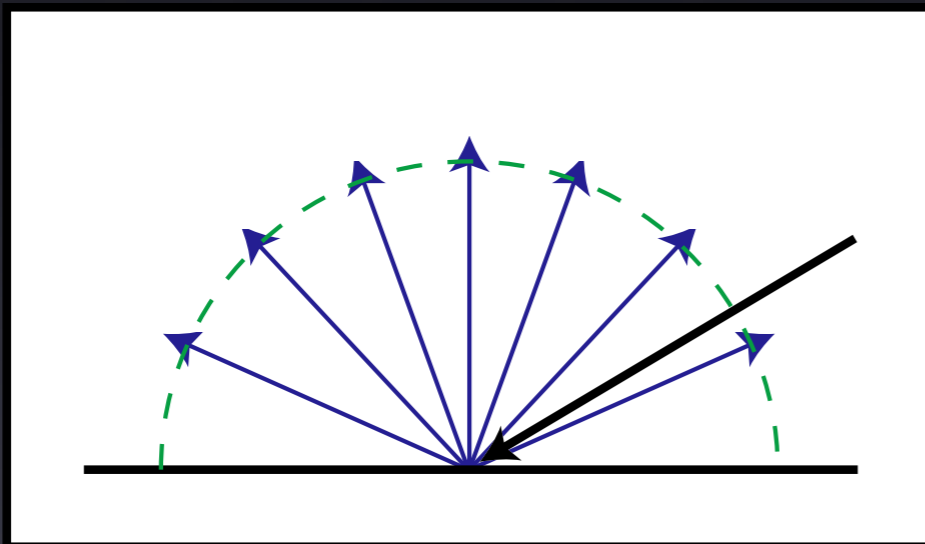
- Ward and Heckbert '92
- Křivánek et al. '05a, '05b
- Tabellion and Lamorlette '04
- Tawara et al. '04

# Radiance Caching

Křivánek et al. '05a, '05b

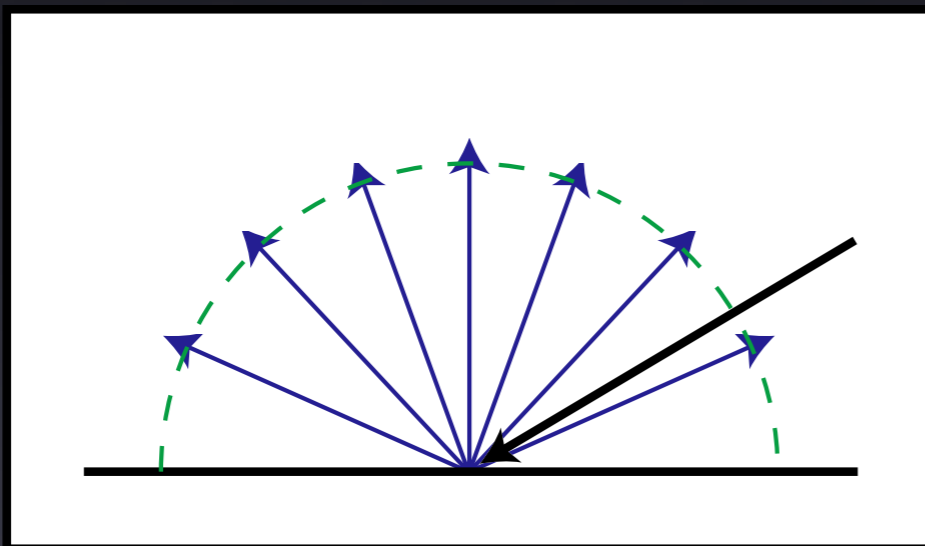
- Extends irradiance caching to glossy surfaces.

# Glossy Surfaces

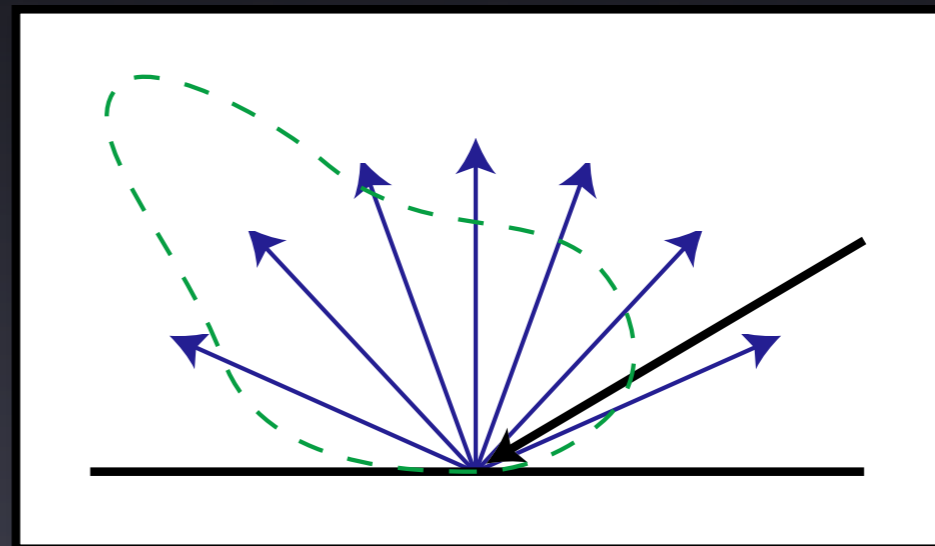


Matte surface

# Glossy Surfaces

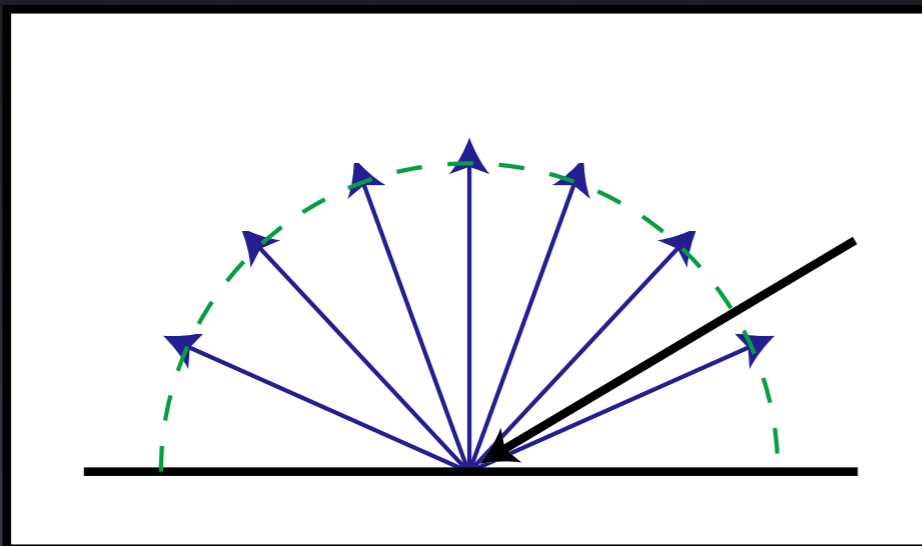


Matte surface

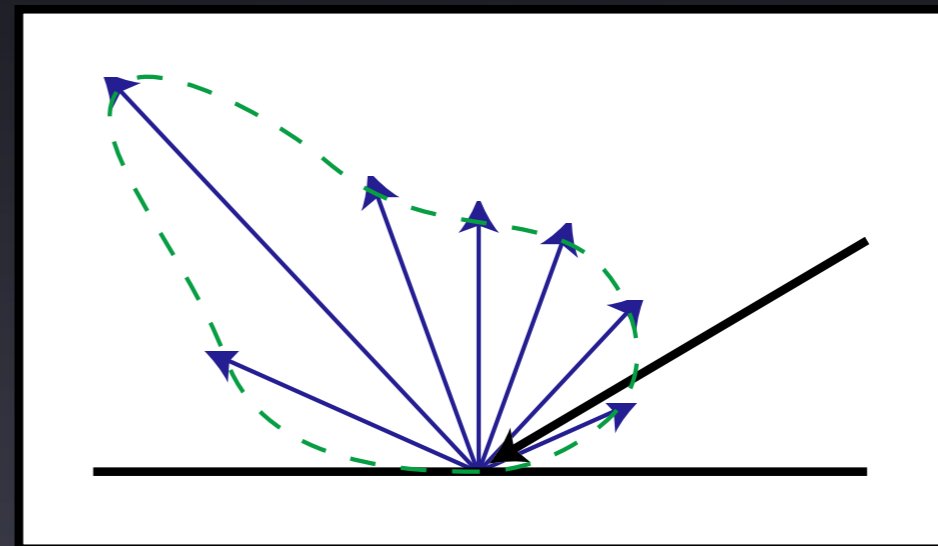


Glossy surface

# Glossy Surfaces

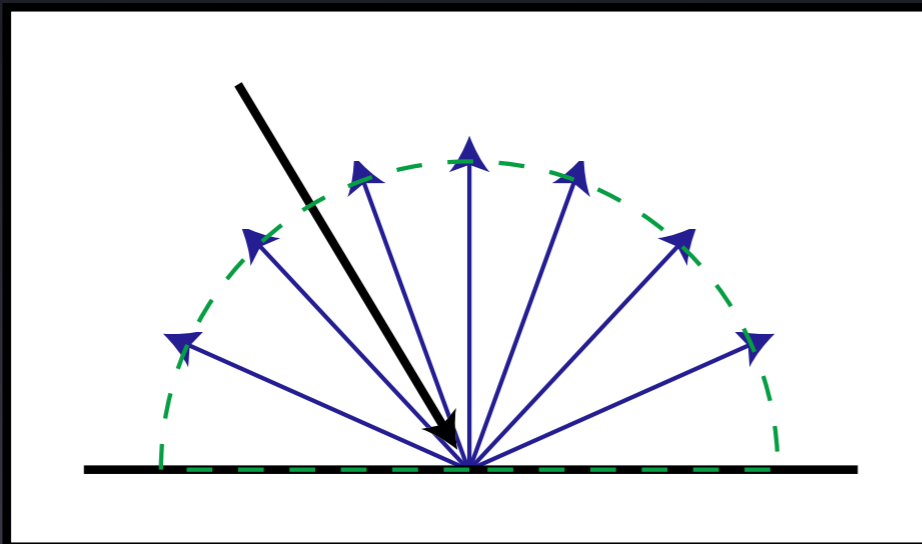


Matte surface

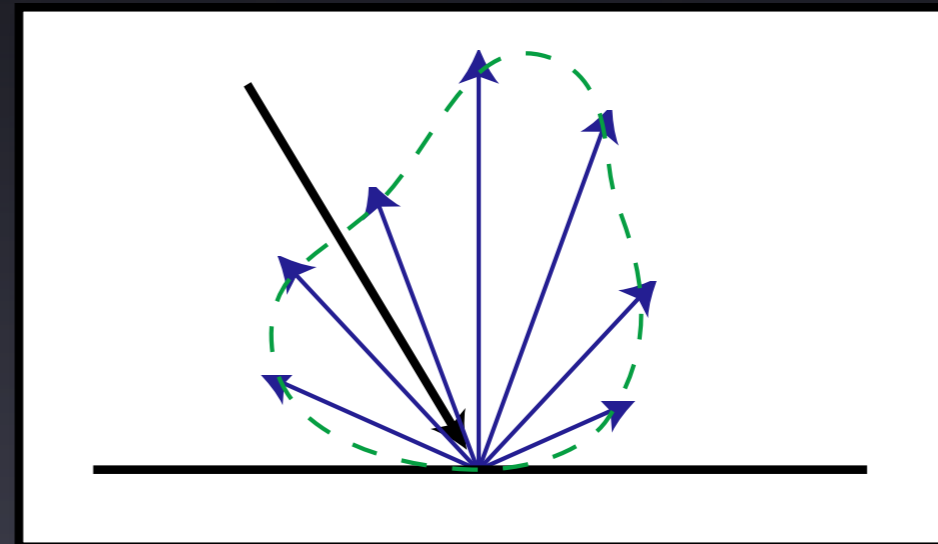


Glossy surface

# Glossy Surfaces

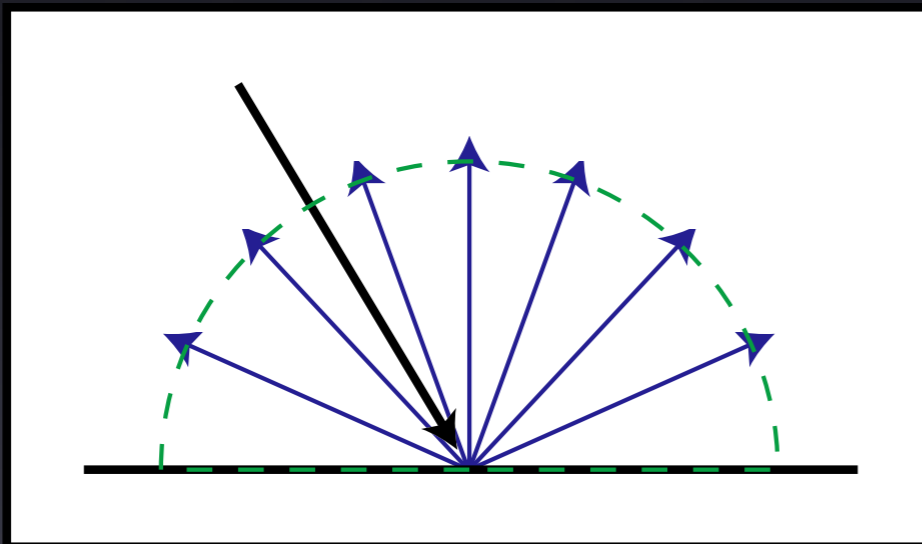


Matte surface

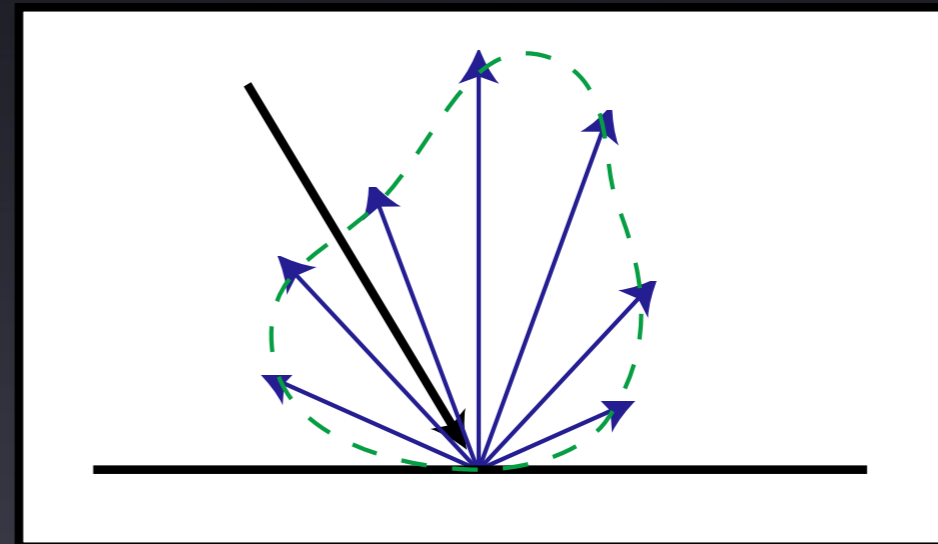


Glossy surface

# Glossy Surfaces



Matte surface



Glossy surface

Bidirectional Reflectance Distribution Function  
BRDF

# Radiance Caching

Křivánek et al. '05a, '05b

- Can no longer cache just the average irradiance value.
- Cache full hemispherical radiance field at sparse locations.

# Radiance Interpolation

Křivánek et al. '05a, '05b

- Interpolate the radiance field from nearby locations.

# Radiance Storage

Krivánek et al. '05a, '05b

- Use spherical harmonics (SH) or hemispherical harmonics (HSH).
- Generalization of Fourier series onto spherical and hemispherical domains.
- Can efficiently approximate smooth functions with just a few numbers.

# (H)SH

Křivánek et al. '05a, '05b

- Projecting a function,  $s$ , onto the (H)SH basis functions results in a set of coefficients:

$$\psi_l^m = \int_0^{2\pi} \int_0^{\frac{\pi}{2}} s(\theta, \phi) H_l^m(\theta, \phi) \sin \theta \, d\theta \, d\phi$$

# (H)SH

Křivánek et al. '05a, '05b

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$$\psi_l^m = \int_0^{2\pi} \int_0^{\frac{\pi}{2}} s(\theta, \phi) H_l^m(\theta, \phi) \sin \theta \, d\theta \, d\phi$$

- From these coefficients, we can approximate the original function:

$$s(\theta, \phi) \approx \sum_{l=0}^{n-1} \sum_{m=-l}^l \psi_l^m H_l^m(\theta, \phi)$$

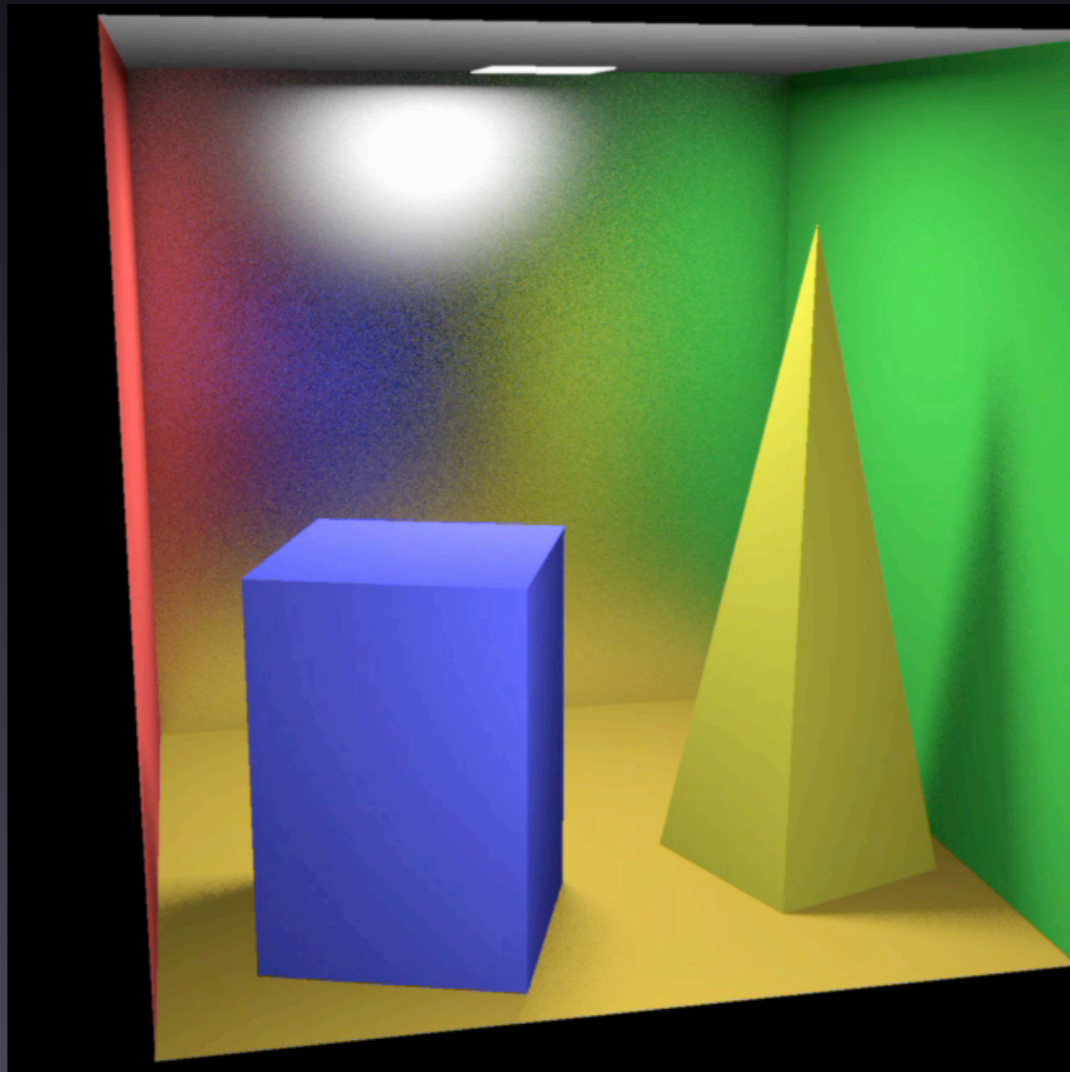
# Outgoing Radiance

Křivánek et al. '05a, '05b

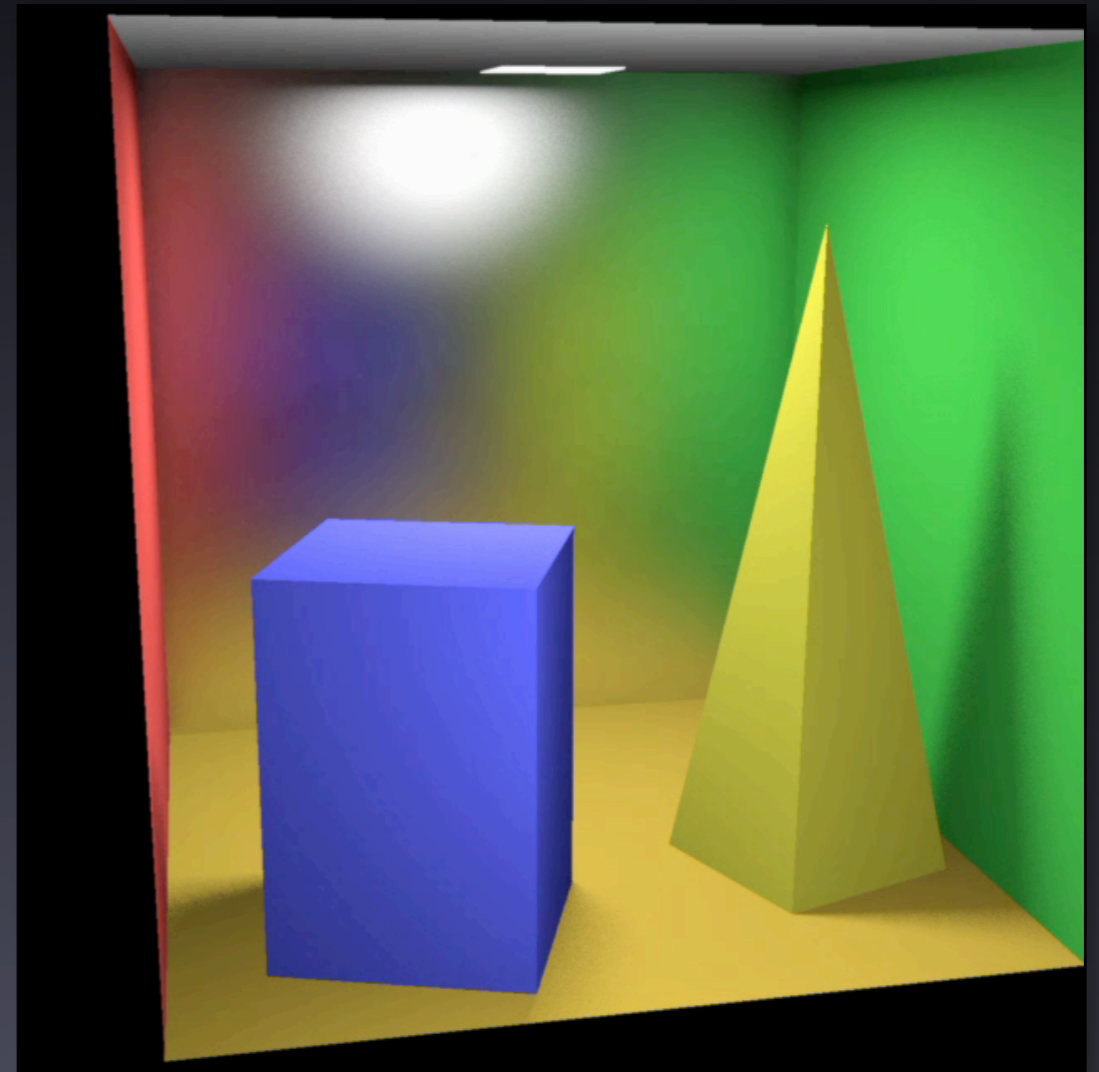
- Incoming radiance must be convolved with the BRDF to attain outgoing radiance.
- If BRDFs are also stored in (H)SH basis this convolution is just a dot product!

# Glossy Surfaces

Křivánek et al. '05a, '05b



Path Tracing



Radiance Caching

from Křivánek et al. '05a, '05b

# Radiance Gradients

Křivánek et al. '05a, '05b

- Improve interpolation quality by storing gradient of incoming radiance field.

# Improvements/Extensions

- Ward and Heckbert '92
- Křivánek et al. '05a, '05b
- **Tabellion and Lamorlette '04**
- Tawara et al. '04

# Simplified Geometry

Tabellion and Lamorlette '04

- Apply approximations to speed up rendering.
- Use simplified scene geometry for indirect illumination.

# Simplified Geometry

Tabellion and Lamorlette '04

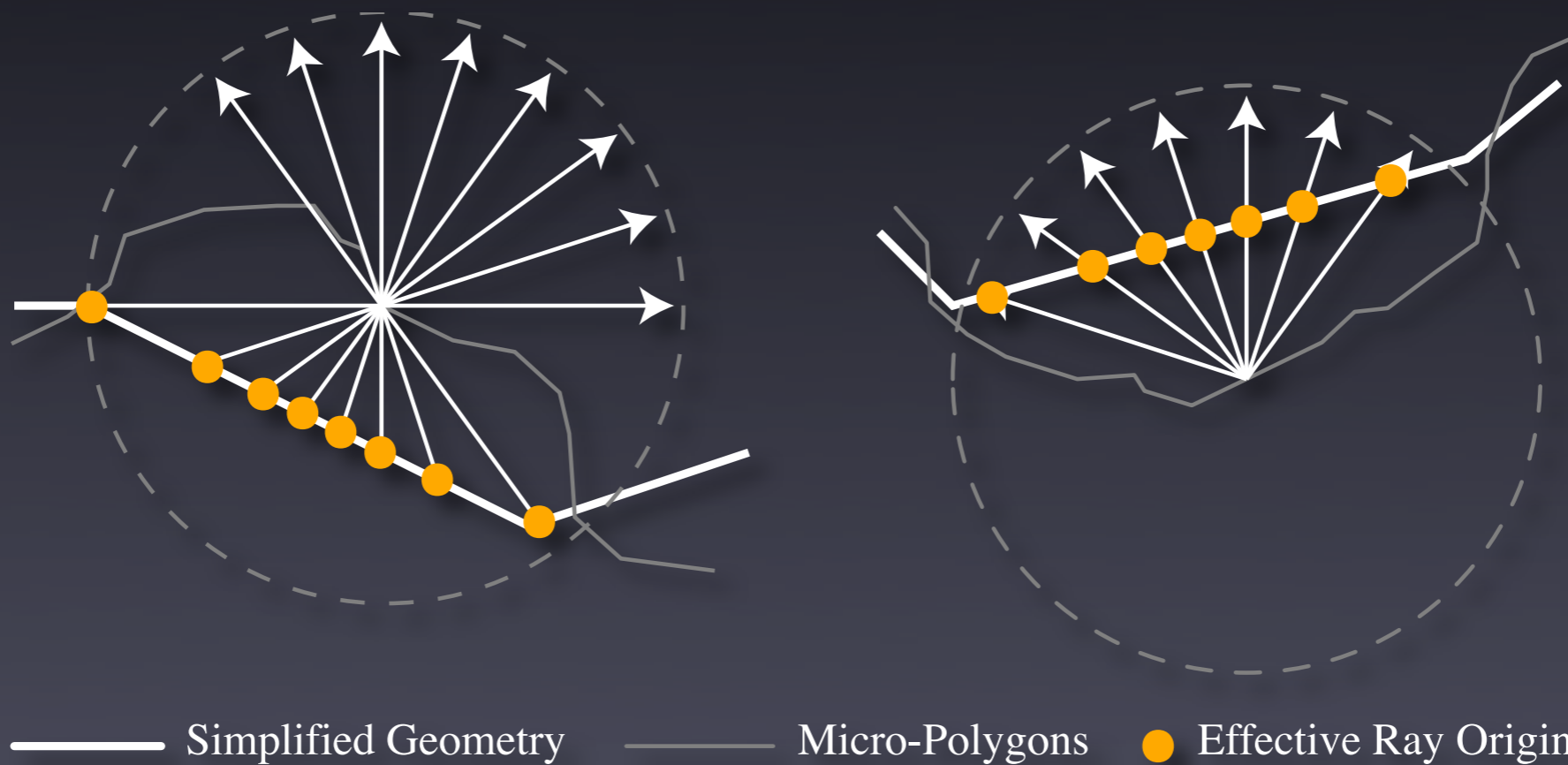


from Tabellion and Lamorlette '04

# Simplified Geometry

Tabellion and Lamorlette '04

- Self intersection:



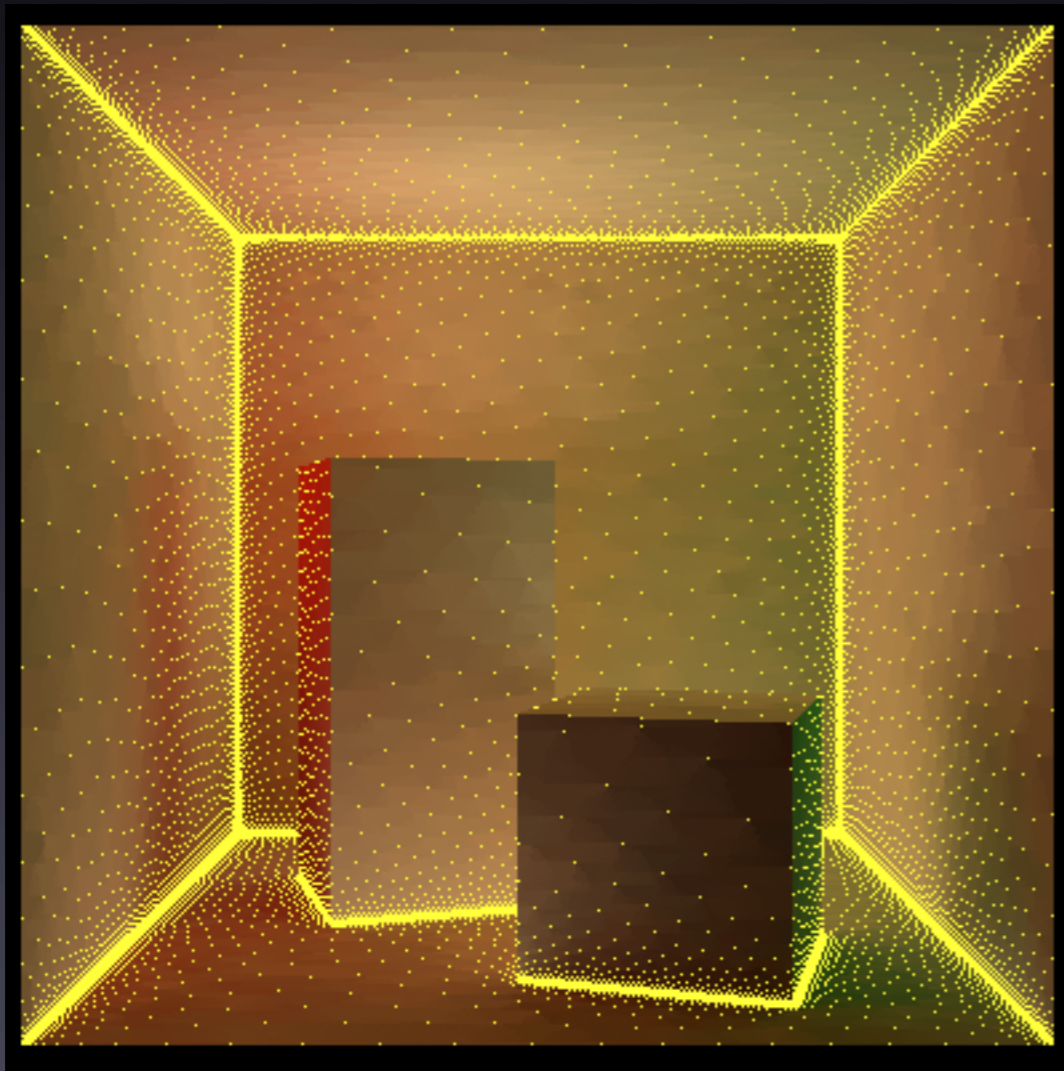
re-created from Tabellion and Lamorlette '04

# Modified Error Metric

Tabellion and Lamorlette '04

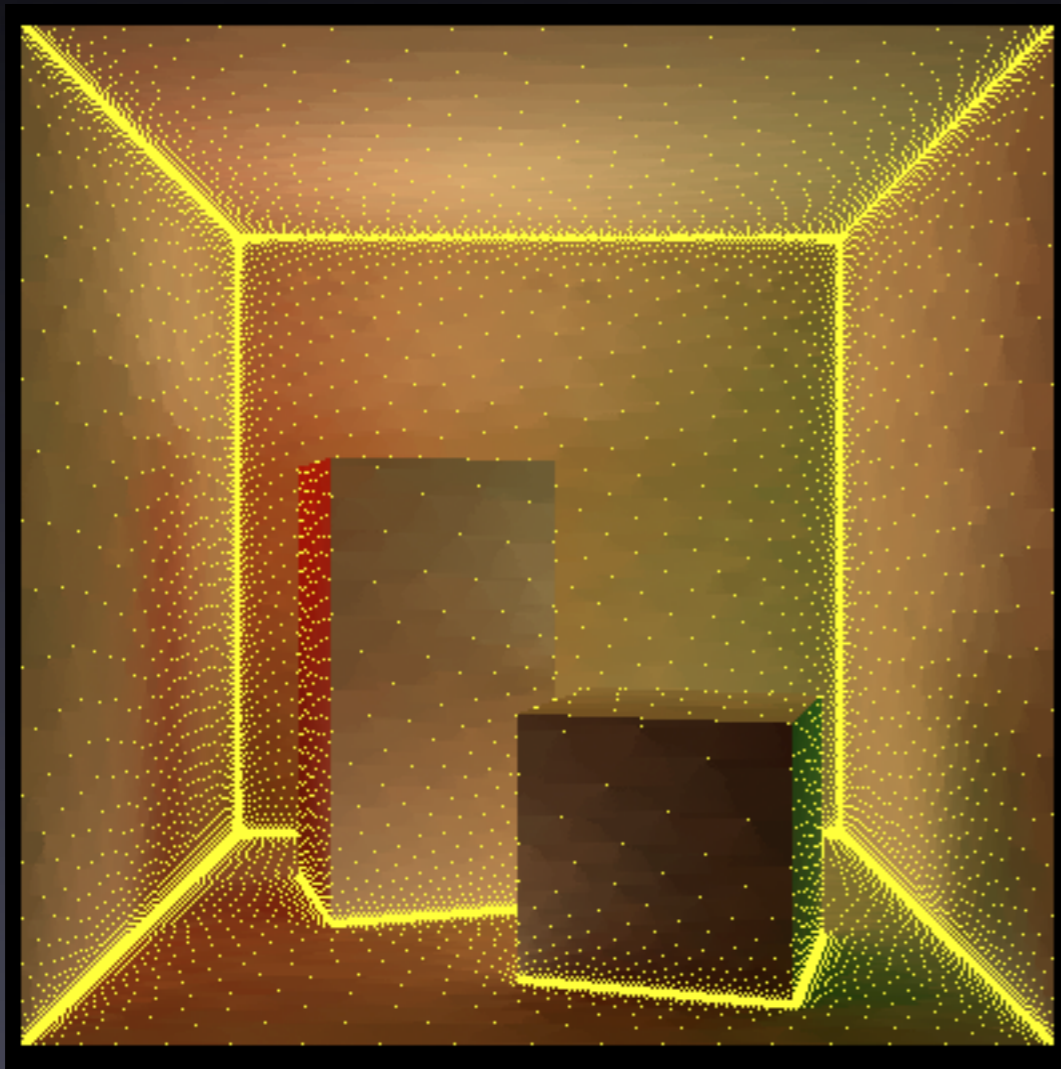
- Introduced new error metric to reduce clumping in corners.

# Over-Sampling/Clumping

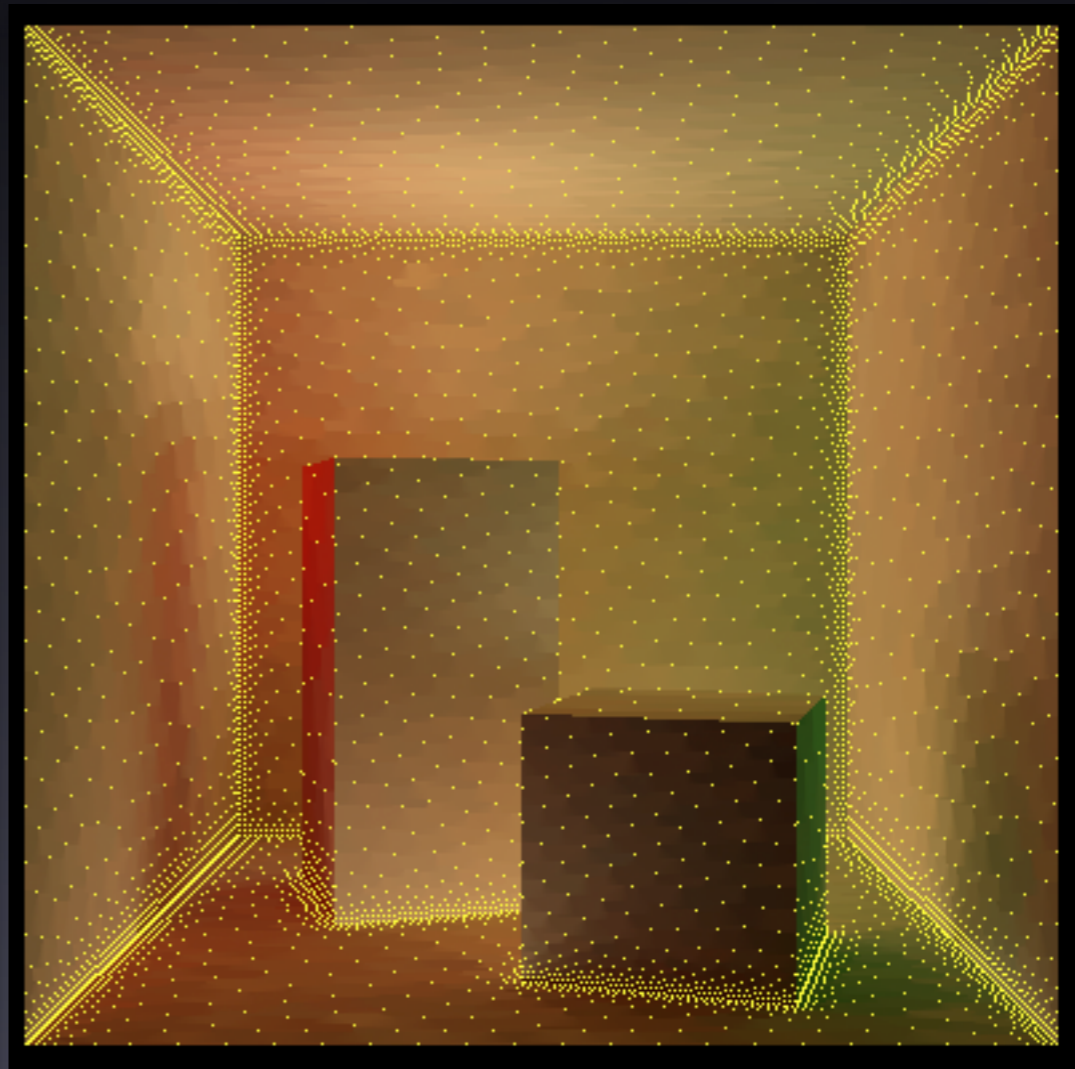


Ward Metric

# Over-Sampling/Clumping

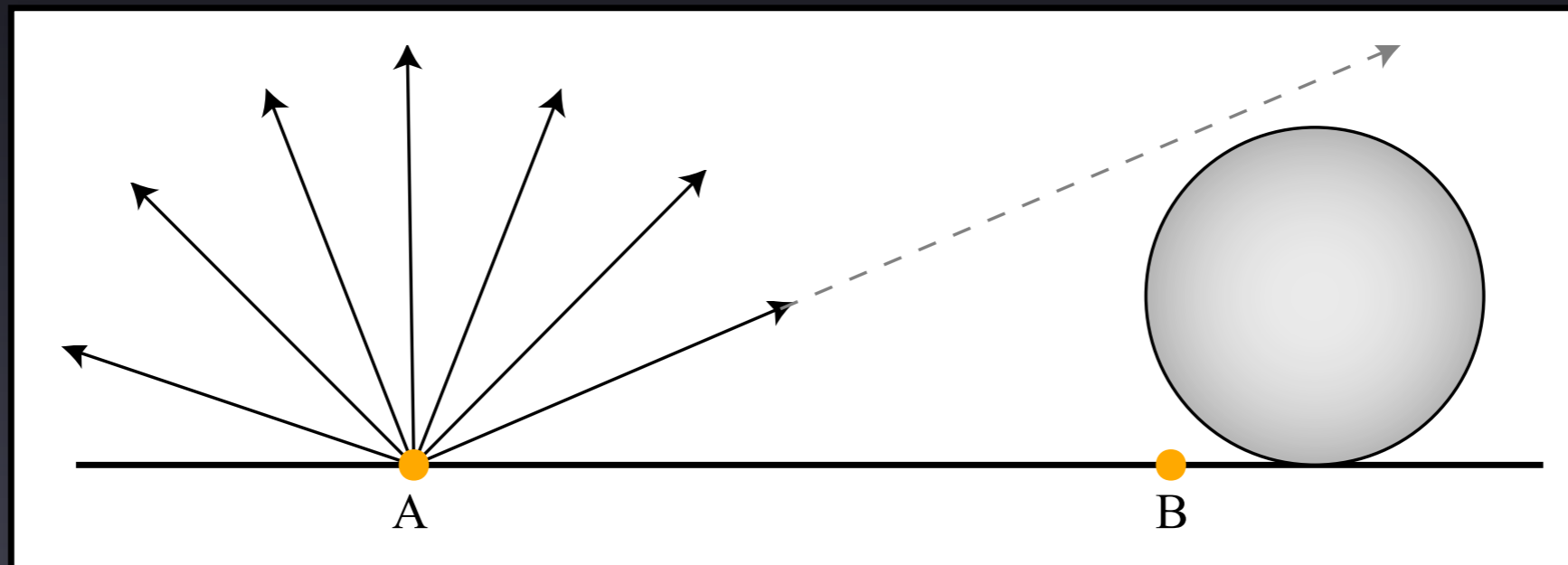


Ward Metric

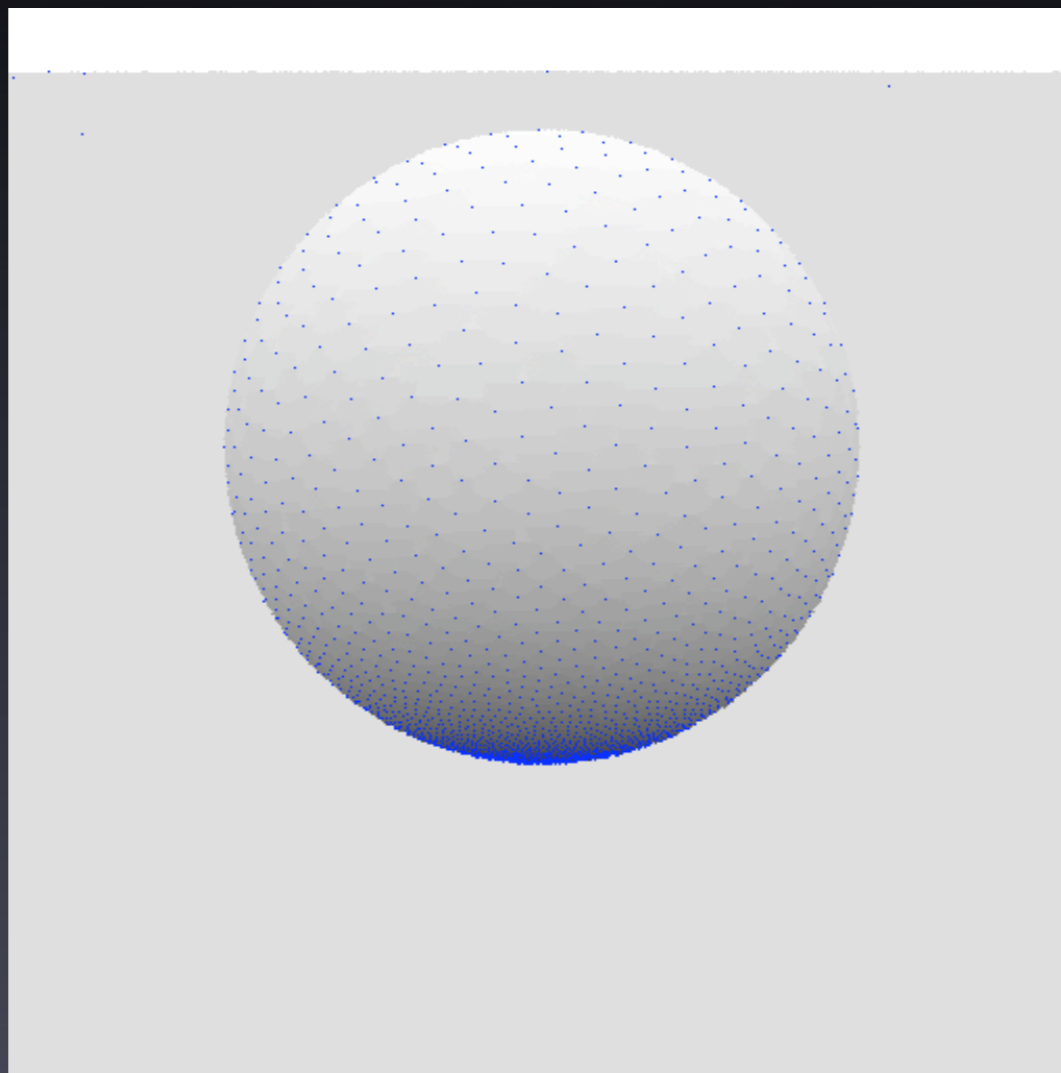


Tabellion Metric

# Under-Sampling

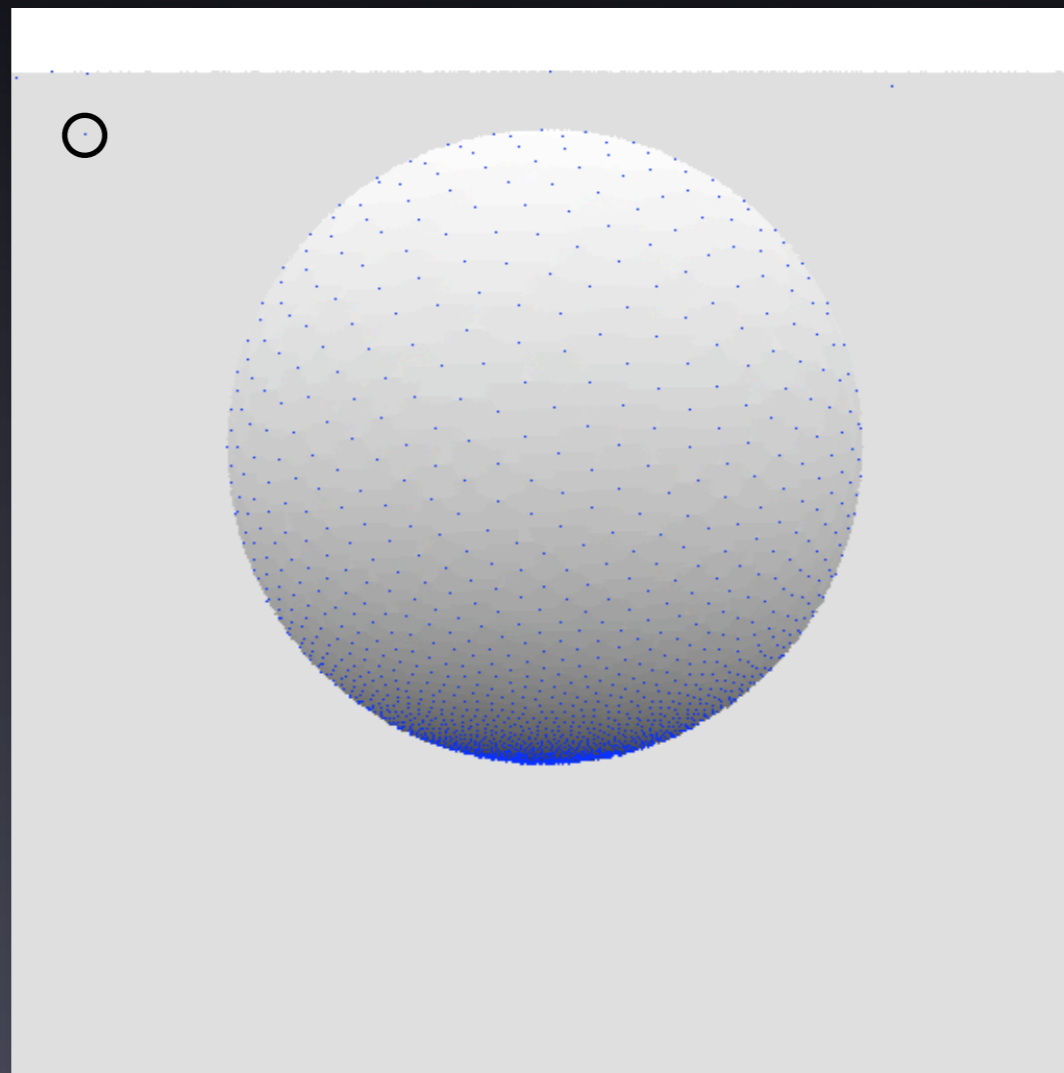


# Under-Sampling



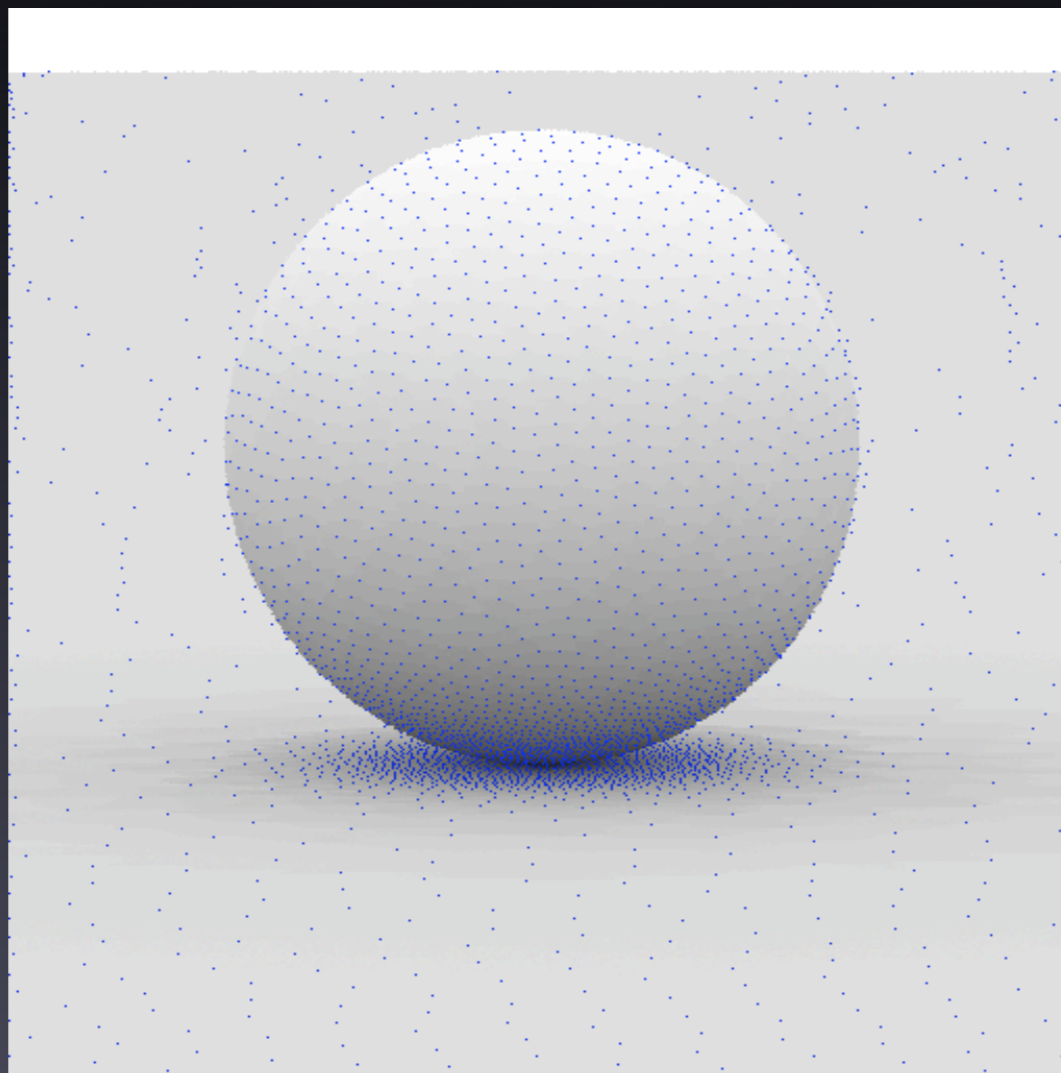
Ward Metric

# Under-Sampling



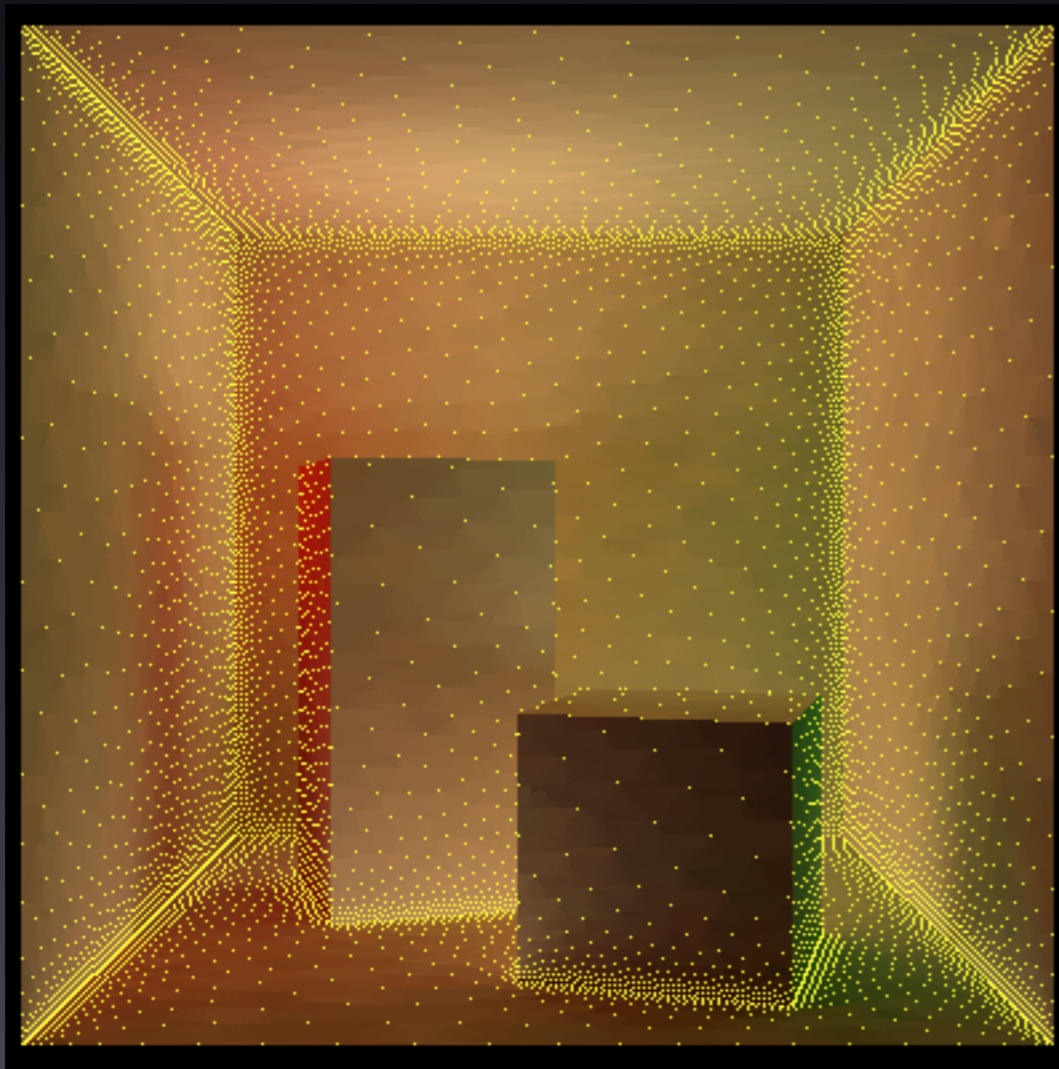
Ward Metric

# Under-Sampling

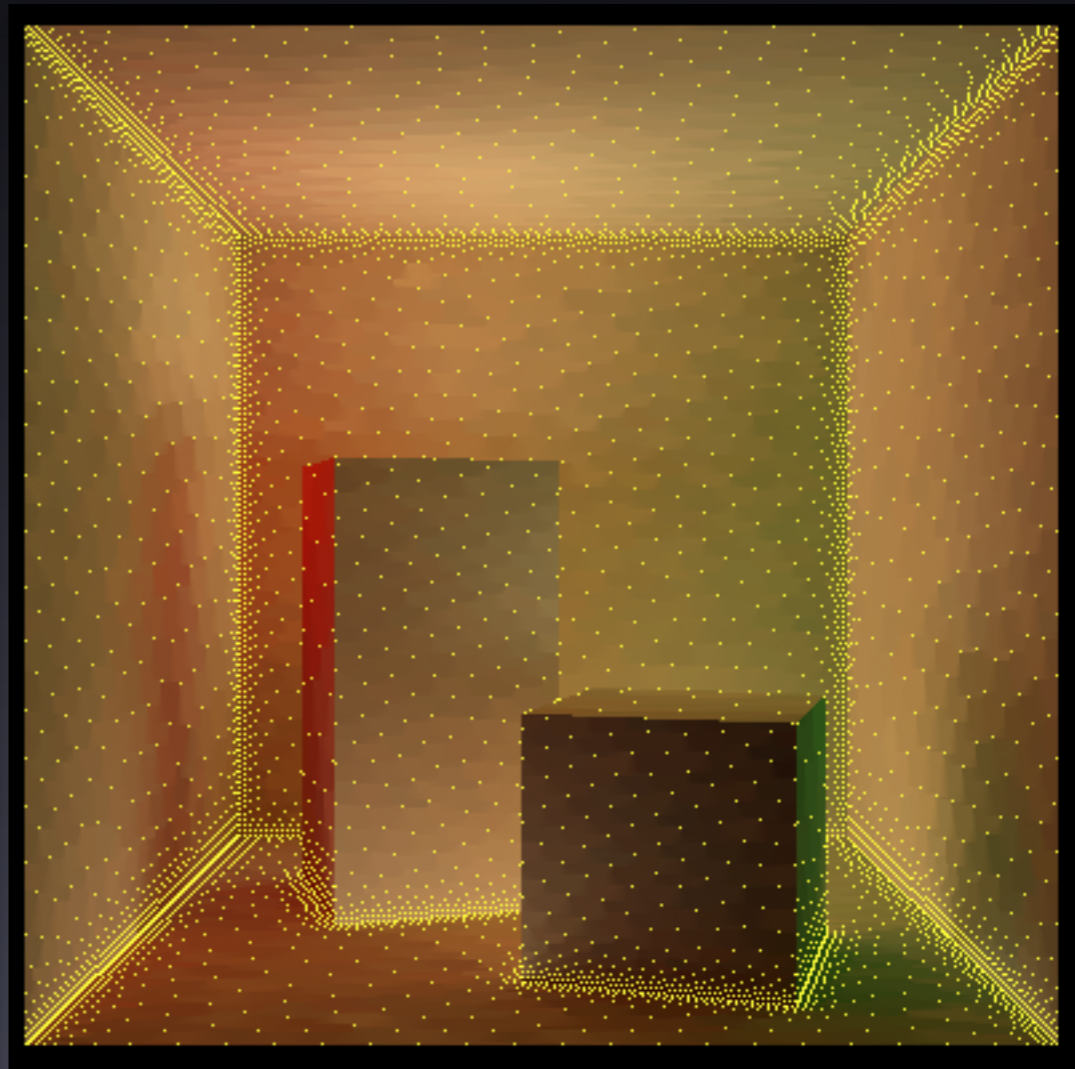


Tabellion Metric

# Modified Ward Metric



Modified Ward Metric



Tabellion Metric

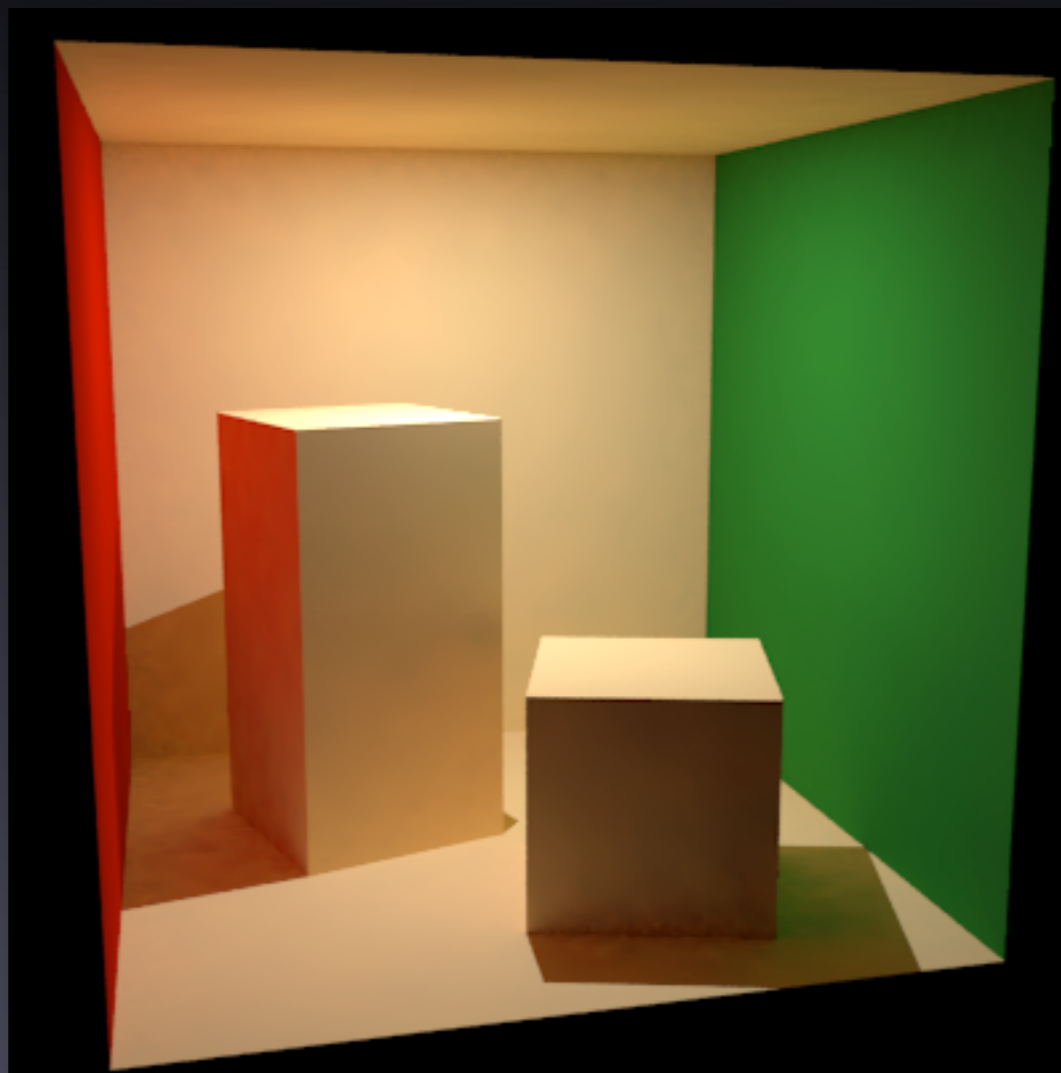
# Improvements/Extensions

- Ward and Heckbert '92
- Křivánek et al. '05a, '05b
- Tabellion and Lamorlette '04
- Tawara et al. '04

# Dynamic Camera



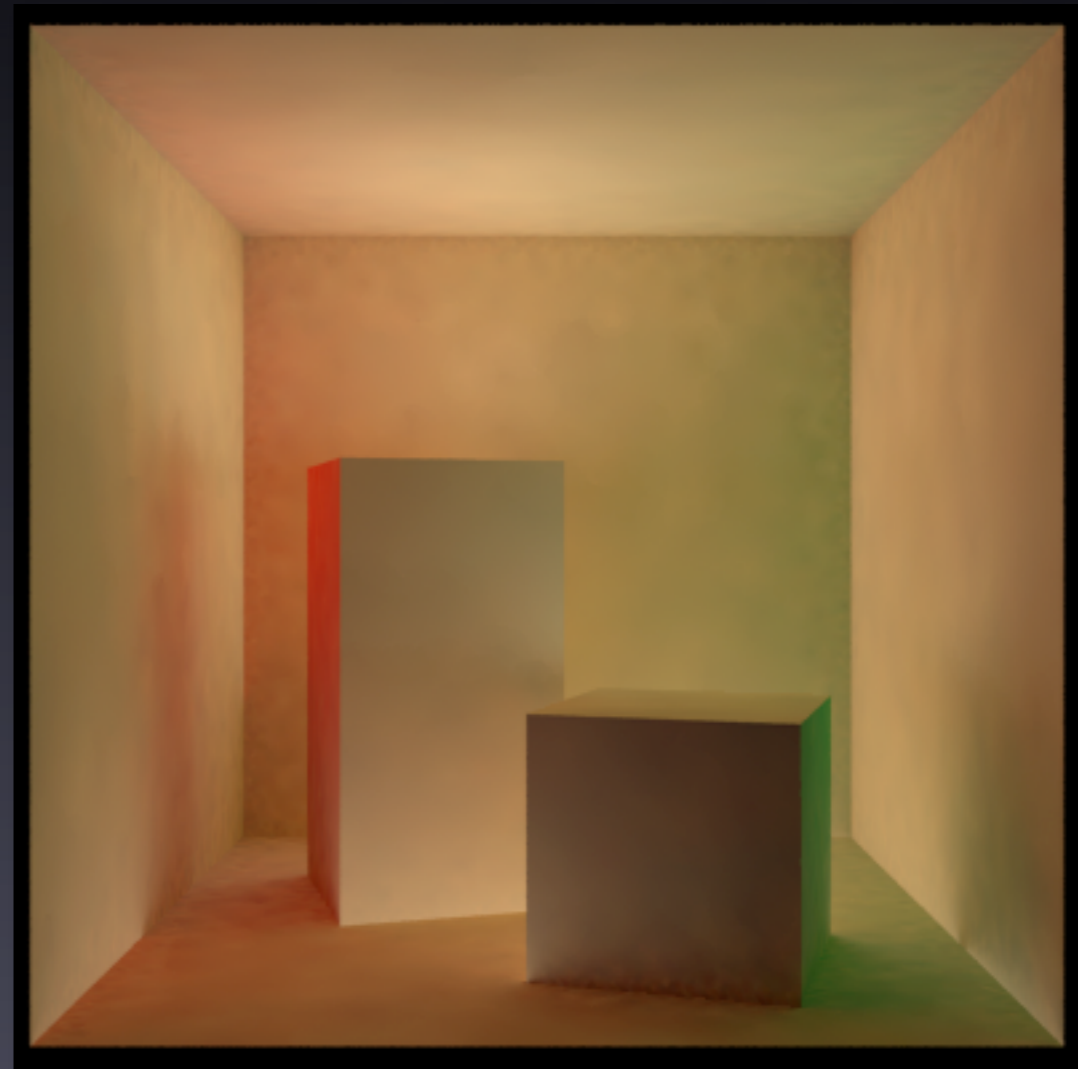
# Dynamic Camera



# Dynamic Objects

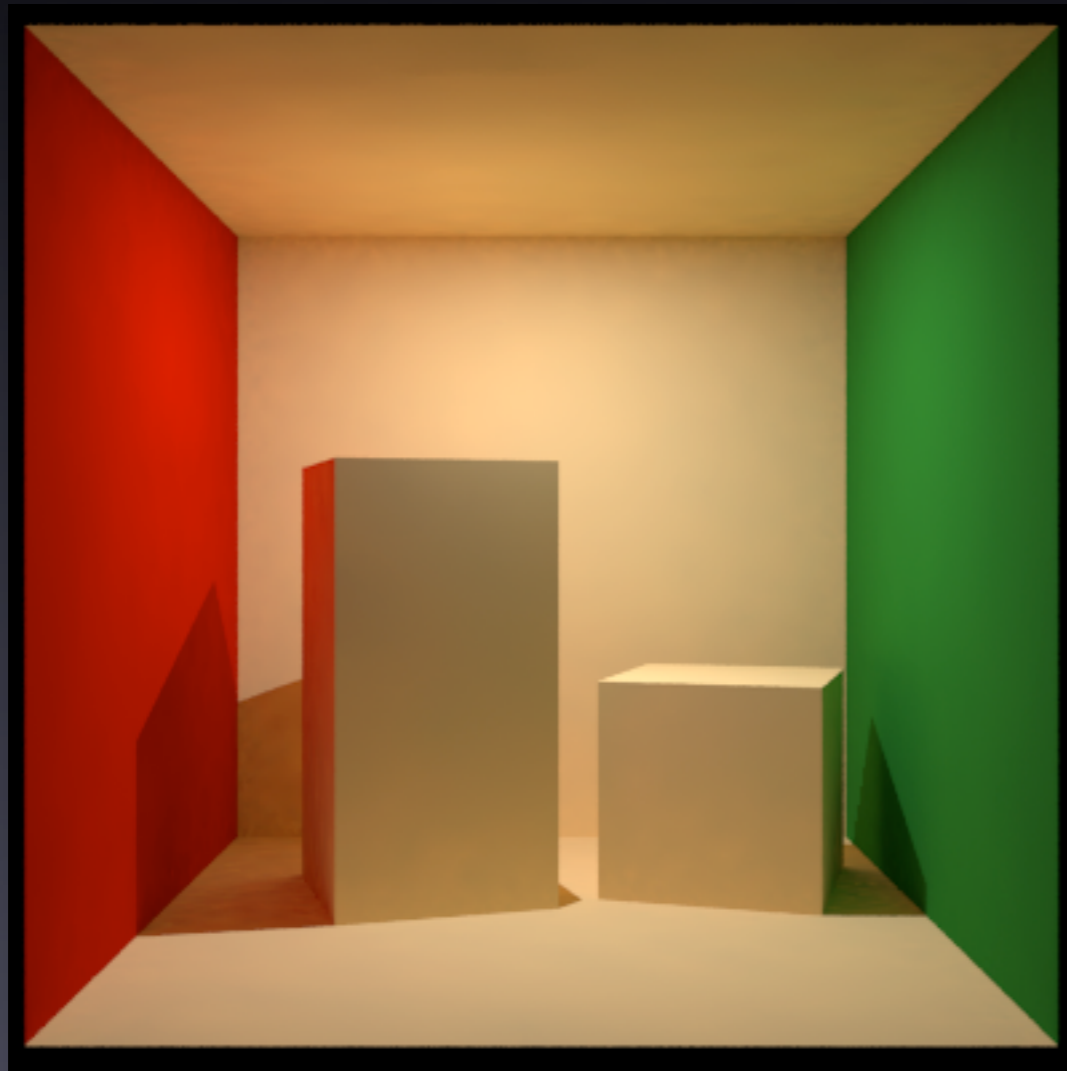


Direct + Indirect

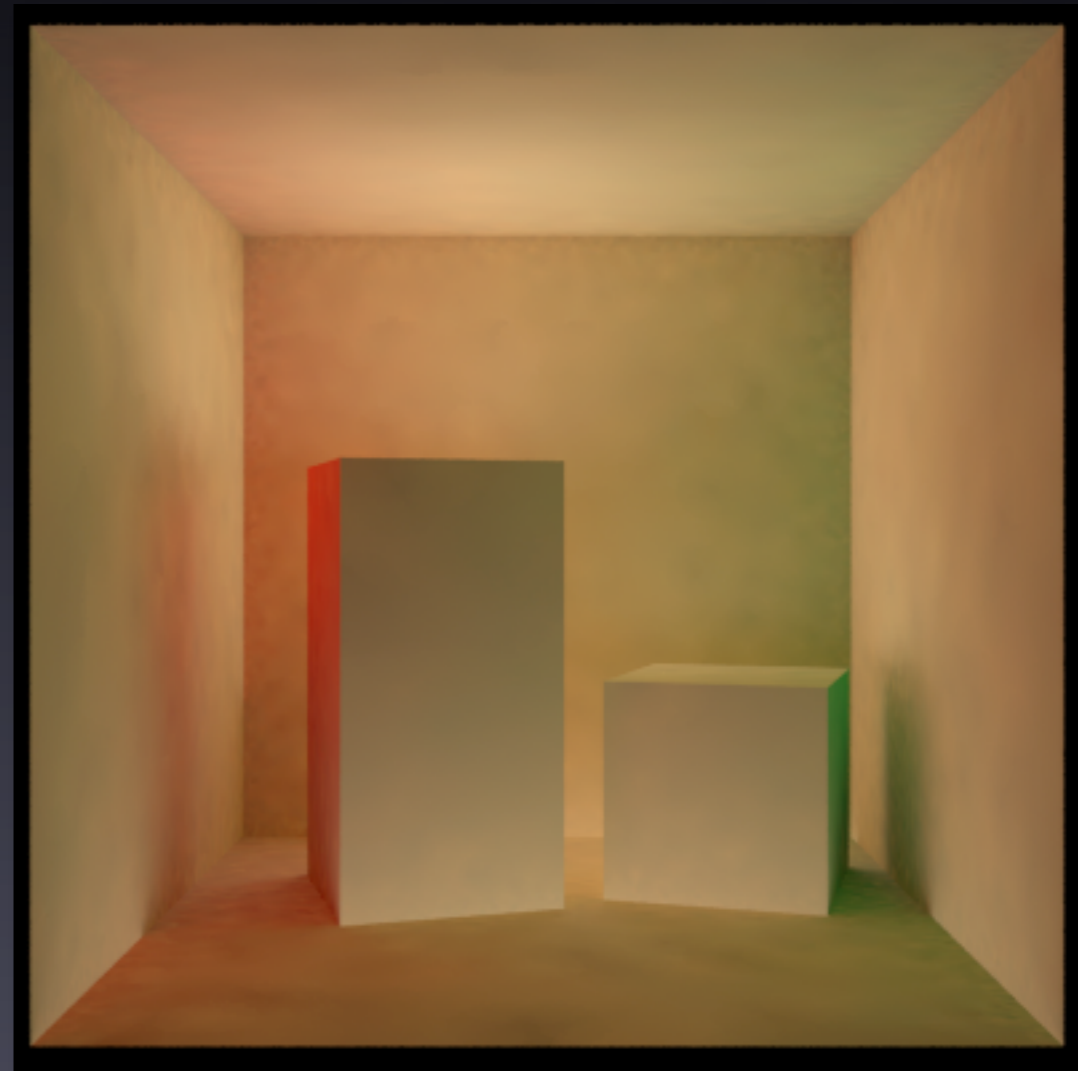


Indirect Illumination

# Dynamic Objects



Direct + Indirect



Indirect Illumination

# Age-based Invalidation

Tawara et al. '04

- Only recompute a small subset of global gather rays.
- Store full stratified radiance field.
- Assign age to each stratum.
- Update a small percentage of oldest strata (10%) for frame.
- Can also give higher probability to strata which “see” moving objects.

# Improvements/Extensions

- Ward and Heckbert '92 - better interpolation
- Křivánek et al. '05a, '05b - glossy surfaces
- Tabellion and Lamorlette '04 - speed
- Tawara et al. '04 - animation
- Yee '00 - speed/perception
- Kato '02 - parallel/distributed computation
- Arikan et al. '05 - speed

Next time:

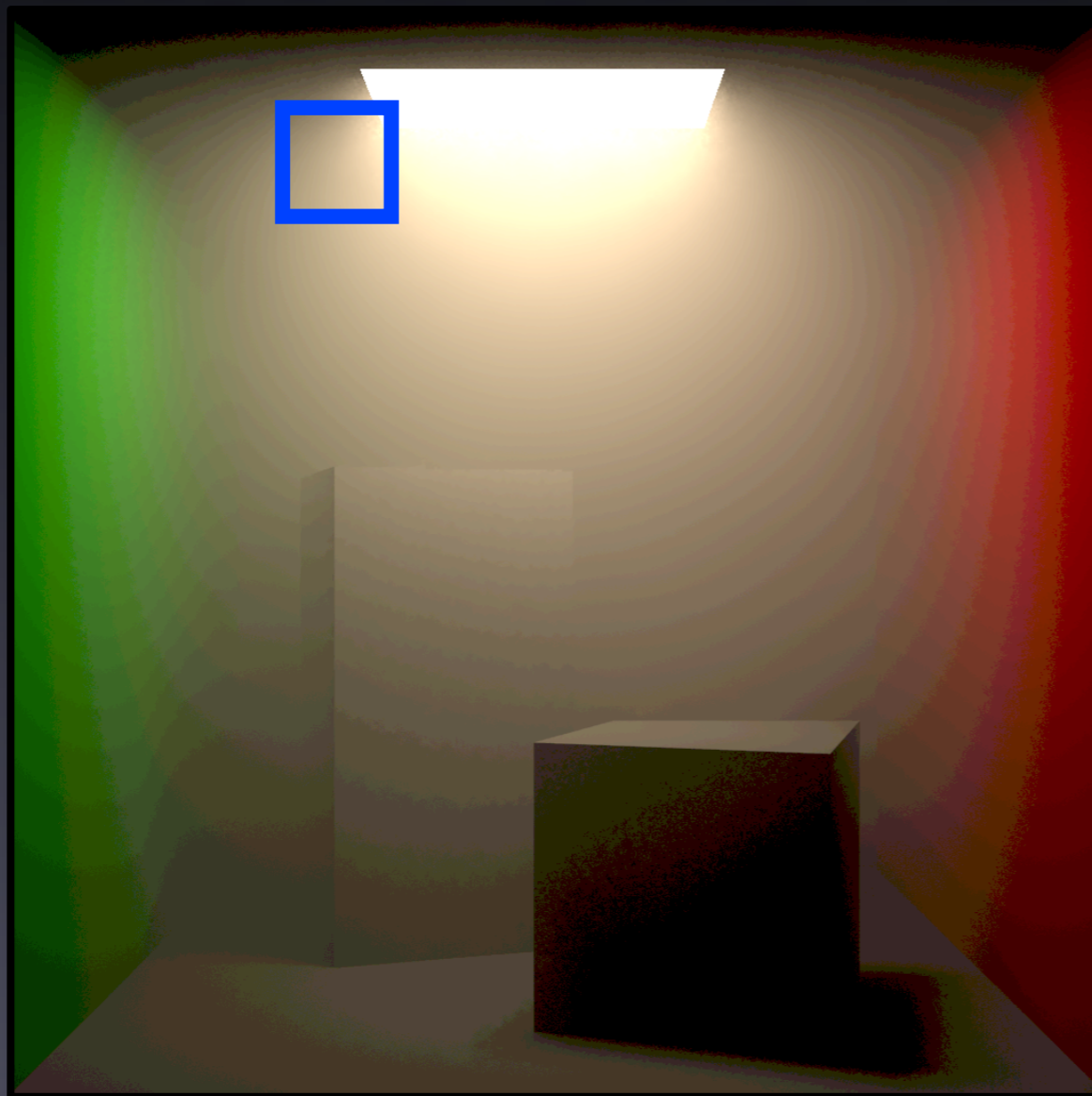
# Next time:

- Participating Media

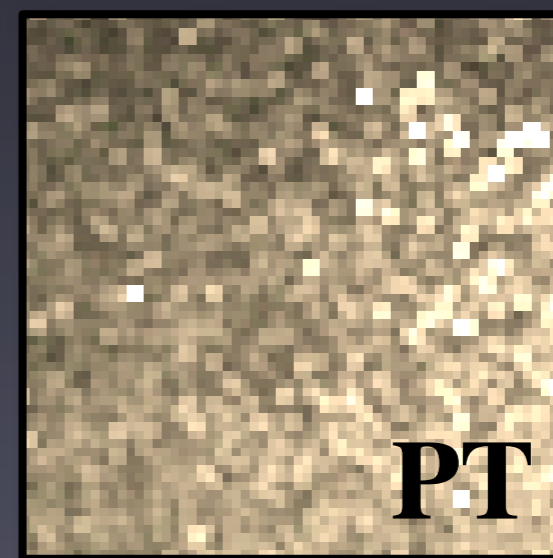
# Participating Media



# Smoky Cornell Box



**RC**



**PT**

# Foggy Road



# Foggy Road



# Questions?